QCD Susceptibilities, Nuclear Saturation and Two-Pion Processes

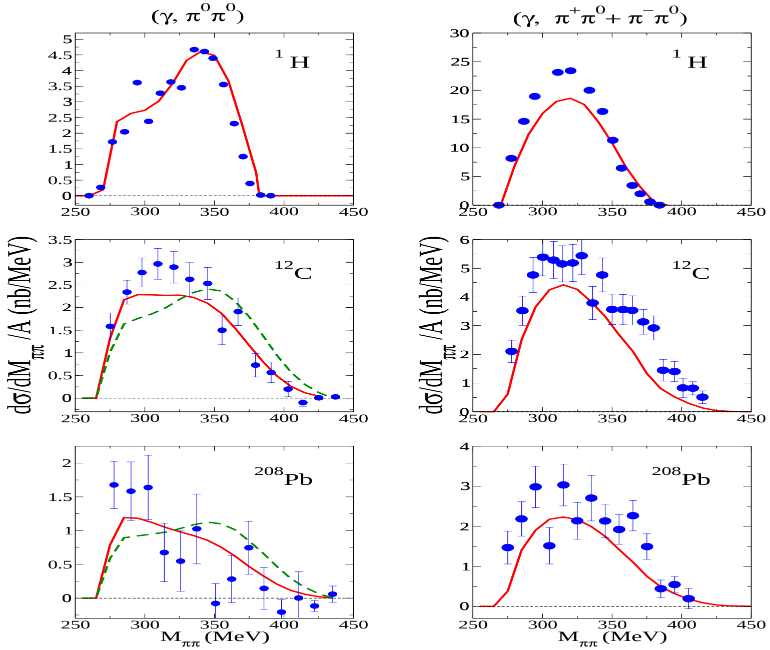
(M. Ericson, P. Guichon, G.C)

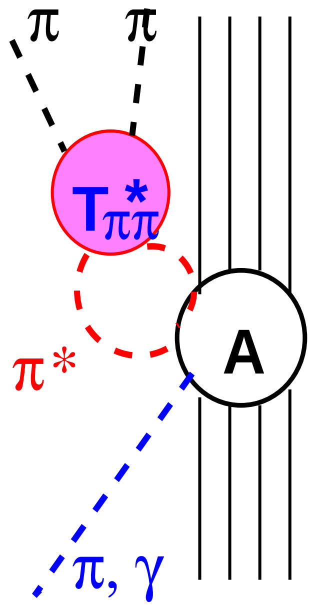
2π PROCESSES AND CHIRAL SYMMETRY

- $A(\gamma, \pi\pi)$ (TAPS)

Downwards shift of the $\pi\pi$ invariant mass distribution in the scalar-isoscalar channel $I = J = 0$

- What is the role of
- Chiral Dynamics
- Chiral restoration ?

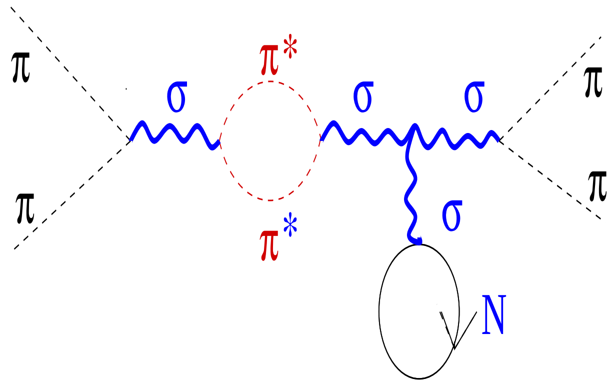




- In-medium $I = J = 0$ $\pi\pi$ interaction

– In-medium pions (Schuck *et. al*)

– Dropping of m_σ, f_π (Hatsuda *et. al*)



DROPPING OF THE SIGMA MASS

– Associated with partial chiral restoration
  Linear sigma model (Hatsuda) : 30% dropping at $\rho = \rho_0$

$$\frac{m_\sigma^{*2}}{m_\sigma^2} = 1 + 3\,\frac{\langle s\rangle}{f_\pi} \qquad \frac{\langle \bar{q}q\rangle^*}{\langle \bar{q}q\rangle_{vac}} = 1 + \frac{\langle s\rangle}{f_\pi} - \frac{\langle \phi_\pi^2\rangle}{2\,f_\pi^2}$$

– But constraints from nuclear matter stability
– Closely related to the $p-h$ contribution to the nuclear scalar susceptibility

IN-MEDIUM MODIFIED TWO-PION EXCITATION

– Closely related to to the 2π contribution to the nuclear scalar susceptibility
– Also linked to chiral restoration

In-medium Scalar and Pseudoscalar susceptibilities

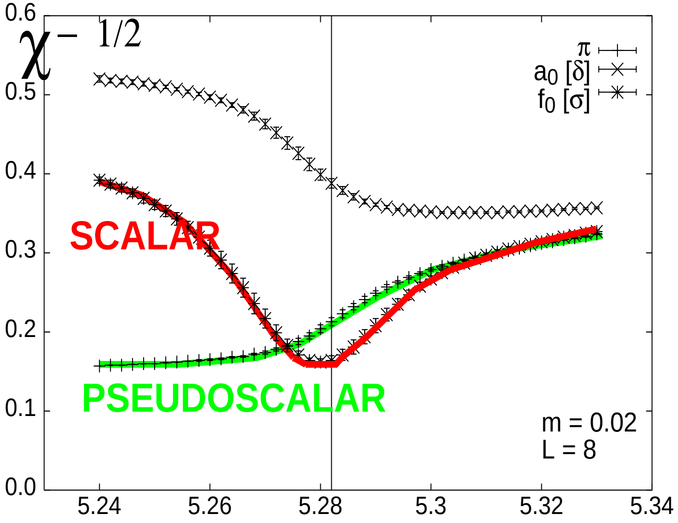
Partial Chiral symmetry restoration ?

FLUCTUATIONS OF THE QUARK CONDENSATE

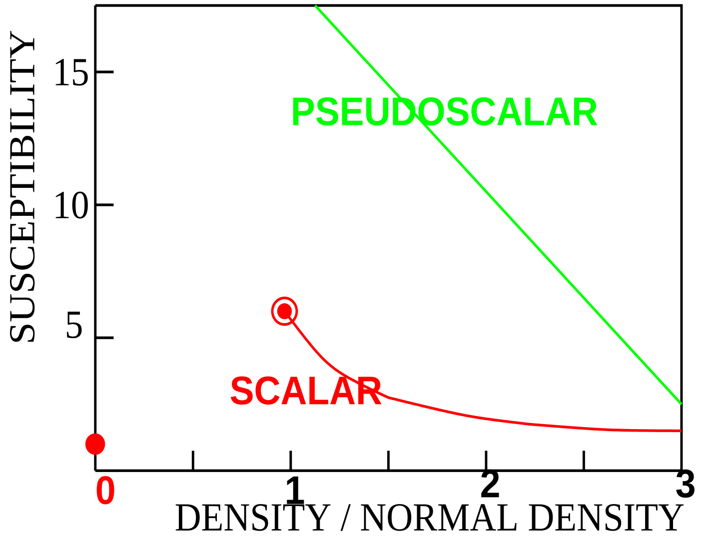
Scalar susceptibility : from the scalar correlator *i.e.* the correlator of the scalar quark density fluctuations

$$\chi_s = \frac{\partial \langle\langle \bar{q}\, q\rangle\rangle}{\partial m} = 2 \int dt\, d\vec{r}\, \langle\langle \delta\, \bar{q}\, q(0,0),\, \delta\, \bar{q}\, q(\vec{r},t)\rangle\rangle$$

Thermal susceptibilty on Lattice (Karsch)



Finite density : effective chiral theory (M. Ericson, G. C)



Chiral Restoration : $\chi_S \rightarrow \chi_{PS}$

SCALAR SUSCEPTIBILITY

$$\chi_S = \frac{\partial \langle \bar{q}q \rangle}{\partial m_q} = 2 \int dt' \, d\mathbf{r}' \, \Theta(t - t') \langle -i \left[\bar{q}\, q(0) \, , \, \bar{q}\, q(\mathbf{r}' \; t') \right] \rangle$$

$$\chi_S = \left(\frac{\partial^2 \omega}{\partial m_q^2} \right)_\mu = Re G_S(\omega = 0, \vec{q} \rightarrow 0) = \int_0^\infty d\omega \left(-\frac{2}{\pi \omega} \right) Im G_S(\omega, \vec{q} = 0)$$

→ A strong contribution of low energy nuclear excitations is expected

→ Linear sigma model $\bar{q}q \longrightarrow \frac{\langle \bar{q}q \rangle_{vac}}{f_\pi} \sigma$: The nuclear susceptibility is related to the in-medium σ propagator

PSEUDOSCALAR SUSCEPTIBILITY

$$\chi_{PS} = 2 \int dt' \, d\mathbf{r}' \, \Theta(t - t') \langle -i \left[\bar{q}\, i\gamma_5 \frac{\tau_\alpha}{2} q(0) \, , \, \bar{q}\, i\gamma_5 \frac{\tau_\alpha}{2} q(\mathbf{r}' \; t') \right] \rangle = \frac{\langle \bar{q}q \rangle(\rho)}{m_q}$$

(using $\partial^\mu \mathcal{A}^\alpha_\mu(x) = 2\, m_q\, \bar{q}\, i\gamma_5 \frac{\tau_\alpha}{2} q(x)$ and soft pion theorem))

→ χ_{PS} behaves like the chiral condensate

LEADING ORDER ESTIMATE

- Grand Potential : $\omega = \int \frac{4\, d^3p}{(2\pi)^3}\,(E_p - \mu)\,\theta\,(\mu - E_p)$

- Chiral condensate :

$$\langle \bar{q}q\rangle(\rho) - \langle \bar{q}q\rangle_{vac} = \frac{1}{2}\left(\frac{\partial\omega}{\partial m_q}\right)_\mu = \frac{1}{2}\frac{\partial M}{\partial m_q}\left(\frac{\partial\omega}{\partial M}\right)_\mu \equiv \frac{\sigma_N}{2\, m_q}\,\rho_S$$

σ_N : nucleon sigma term, ρ_S : scalar density

- Nuclear susceptibility :

$$\chi_S(\rho) = (\chi_S)_{vac} + \rho_S\,\frac{\partial}{\partial m_q}\left(\frac{\sigma_N}{2\, m_q}\right) + \frac{\sigma_N}{2\, m_q}\left(\frac{\partial\rho_S}{\partial m_q}\right)_\mu \equiv \rho_S\,\chi_S^N + \chi_S^{nuclear}$$

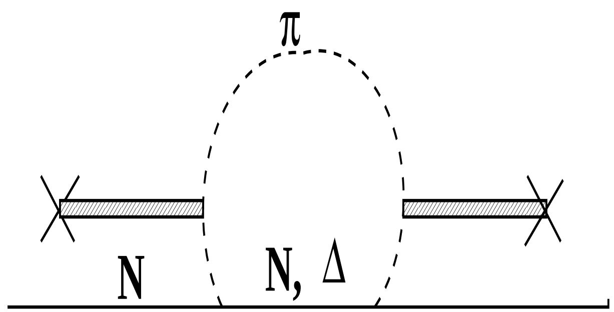
χ_S^N : nucleonic scalar susceptibility

NUCLEONIC CONTRIBUTION : dominated by the pion cloud through the Leading non analytical contribution

$$\chi_S^N = -\frac{2\langle\bar{q}q\rangle_{vac}^2}{f_\pi^4\, m_\pi}\,\frac{9}{64\pi}\,\left(\frac{g_A}{f_\pi}\right)^2\,\left(\frac{\Lambda}{\Lambda+m_\pi}\right)^4$$

LNAC : $\rho_S\,\chi_S^N = 0.08\,(\chi_{PS})_{vac}$

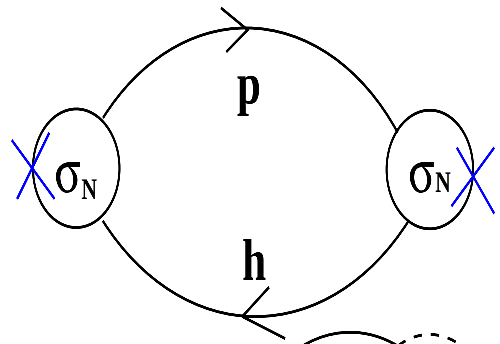
With Δ +FF+Pauli : $\rho_S\,\chi_S^N \simeq 0.04 - 0.05\,(\chi_{PS})_{vac}$



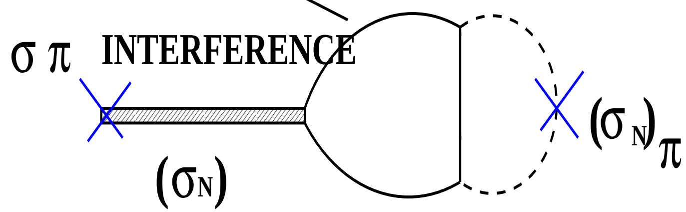
NUCLEAR (p-h) EXCITATION CONTRIBUTION : Compressibility K

$$\chi_S^{nuclear} = \frac{\sigma_N^2}{2\,m_q^2}\left(\frac{\partial\rho_S}{\partial M}\right)_\mu \equiv \frac{\sigma_N^2}{2\,m_q^2}\left(-\frac{2\,p_F\,M_N^2}{\pi^2}\right)$$

$$\left(-\frac{2\,p_F\,M_N^2}{\pi^2}\right) = \Pi_{ph}^0(\omega=0,\vec{q}\to 0) = -\left(\frac{9\,\rho}{K}\right)_{\rho=\rho_0}$$



Sigma model : $\sigma_N = M_N\,\frac{m^2\pi}{m_\sigma^2} + \sigma_N^{(\pi)}$



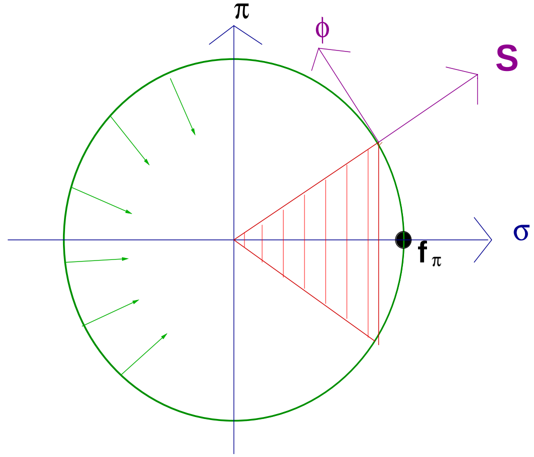
$$\chi_S^{nuclear} = 0.35\,(\chi_{PS})_{vac}$$

CHIRAL EFFECTIVE THEORY

- Order parameter : $\mathcal{M} = \left(\frac{\bar{\psi}\,\psi}{2}\right) + i\vec{\tau}\cdot\left(i\,\bar{\psi}\,\gamma_5\,\frac{\vec{\tau}}{2}\,\psi\right) = \boxed{\sigma + i\vec{\tau}\cdot\vec{\pi}}$

- From Cartesian to Polar coordinates

$$\sigma + i\vec{\tau}\cdot\vec{\pi} = \boxed{(f_\pi + s\,)\,\exp\left[i\frac{\vec{\tau}\cdot\vec{\phi}}{f_\pi}G\left(\frac{\phi^2}{f_\pi^2}\right)\right]}$$



– Pion ϕ : phase fluctuation

– Scalar field $s = S - f_\pi$: amplitude fluctuation

– $\frac{\langle\bar{q}q\rangle_{medium}}{\langle\bar{q}q\rangle_{vacuum}} = 1 - \frac{\langle\phi_\pi^2(\rho)\rangle}{2\,f_\pi^2} - \frac{|\langle s(\rho)\rangle|}{f_\pi}$

- Effective Lagrangian

$$\mathcal{L} = i\bar{N}\gamma^\mu\partial_\mu N - M_N^*(s)\bar{N}N + \frac{1}{2}\partial^\mu s\partial_\mu s - \frac{m_\sigma^2 - m_\pi^2}{8f_\pi^2}\left(s^2 + 2f_\pi s + \frac{2f_\pi^2 m_\pi^2}{m_\sigma^2 - m_\pi^2}\right)^2$$
$$+ \mathcal{L}_\omega + \mathcal{L}_{\pi NN}^{pwave} + \mathcal{L}_{\pi\pi} + \mathcal{L}_{WT}$$

$M_N^*(s) = M_N\left(1 + \frac{s}{f_\pi}\right)$ BUT NO MATTER STABILITY

MATTER STABILITY IN CHIRAL EFFECTIVE THEORIES

"Shifted" vacuum with chiral order parameter $\bar{S} < f_\pi$

Energy density : $\epsilon(\rho, \bar{S}) = \sum_{p<p_F} \sqrt{p^2 + M_N^*(\bar{S})} + V(\bar{S}) + C_V\,\rho^2$

- NJL (Bentz, Thomas, Birse) : Nucleon $\equiv qq - q$ state. $M_q^* = g_q\,\bar{S}$
- Chiral QHD : $S = f_\pi + s \equiv$ chiral invariant scalar field : $M_N^* = g_{sNN}\,\bar{S}$

$g_{sNN}^*(\bar{S}) = \frac{\partial M_N^*}{\partial \bar{S}}$ drops Needed to stabilize nuclear matter

– NJL : Infrared cutoff : simulate confinement
– QMC : Polarization of the quark WF : nucleon structure

Nuclear saturation vs Nucleon structure and QCD (lattice)

- Include the scalar response κ_{NS} of the nucleon to a scalar field

$$M_N^* = M_N \left(1 + \frac{\bar{s}}{f_\pi}\right) + \frac{1}{2}\kappa_{NS}\,\bar{s}^2$$

- Minimization : $\frac{\partial \varepsilon}{\partial \bar{s}} = g_S^*\,\rho_S + V'(\bar{s}) = 0, \quad g_S^*(\bar{s}) = \frac{\partial M_N^*}{\partial \bar{s}}$ drops with ρ

- In-medium sigma mass : $m_\sigma^{*2} = \frac{\partial^2 \varepsilon}{\partial \bar{s}^2} = V''(\bar{s}) + \kappa_{NS}\,\rho_S$

- χ_S related to in-medium sigma propagator dressed by ph excitations

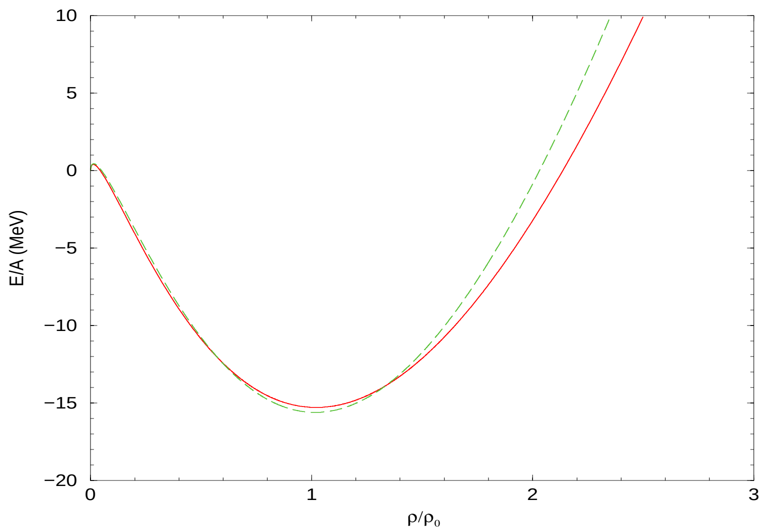
$$\chi_S = -2\,\frac{\langle \bar{q}q\rangle_{vac}^2}{f_\pi^2}\left(-\frac{1}{m_\sigma^{*2}} + \frac{1}{m_\sigma^{*2}}\,\Pi_{SS}(0)\,\frac{1}{m_\sigma^{*2}}\right)$$

$\Pi_{SS}(0)$ is the full scalar polarization propagator (related to K)

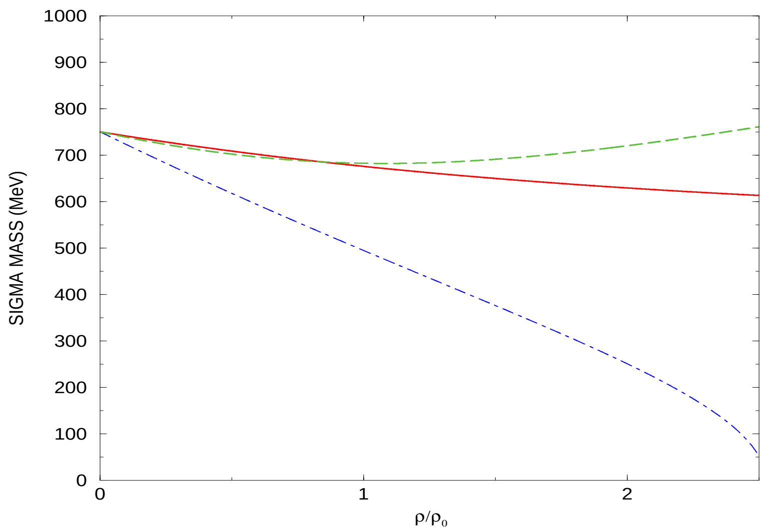
$$\Pi_{SS}(0) = g_S^{*2}\,\frac{M_N^*}{E_F^*}\,\Pi_0(0)\left[1 - \left(\frac{g_\omega^2}{m_\omega^2}\,\frac{E_F^*}{M_N^*} - \frac{g_S^{*2}}{m_\sigma^{*2}}\,\frac{M_N^*}{E_F^*}\right)\Pi_0(0)\right]^{-1}$$

- Parameters : m_σ ($\pi\pi$ phase shifts), g_ω and κ_{NS}

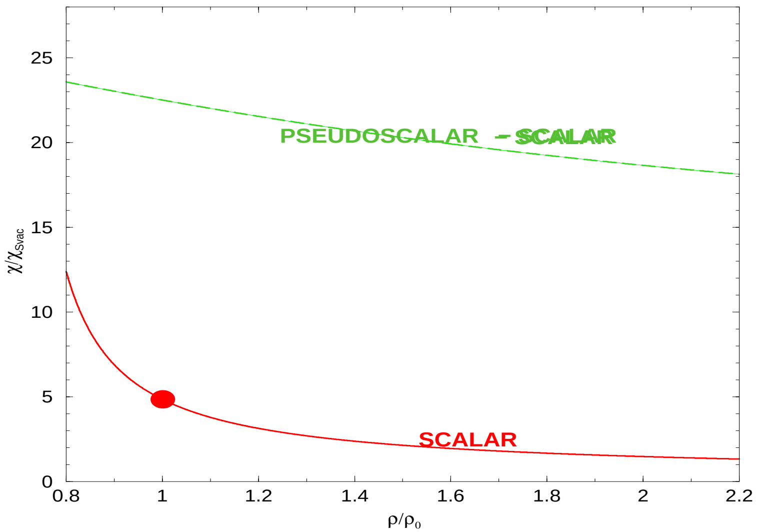
- 2 SETS OF PARAMETERS



- IN-MEDIUM SIGMA MASS



- NUCLEAR SUSCEPTIBILITY



- Nucleon structure effect compensates the chiral dropping !
- κ_{NS} Probably too large

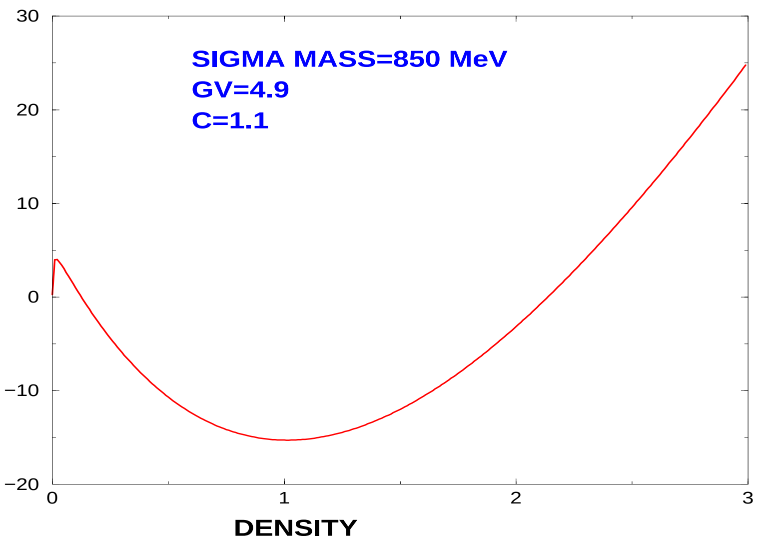
$$\chi_{PS} = (\langle \bar{q}q \rangle(\rho, 0) + m_q \chi_S) \,/\, m_q$$

- $\rightarrow$ Introduce pion loops (CHIPT or pionic correlation energy)

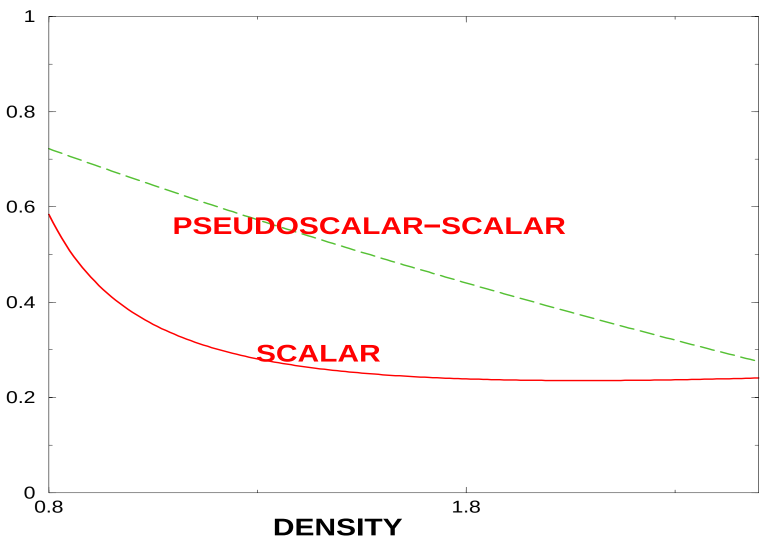
• WITH PIONIC CONTRIBUTION TO THE NUCLEON SIGMA TERM AND PION FOCK TERM (PRELIMINARY)

$$\chi_S = -2\,\frac{\langle\bar{q}q\rangle^2_{vac}}{f_\pi^2}\left(-\frac{1}{m_\sigma^{*2}} + \frac{1}{m_\sigma^{*2}}\left(\frac{\sigma_N^{(\pi)} + \sigma_N^{(\sigma)}}{\sigma_N^{(\sigma)}}\right)^2_{eff} \Pi_{SS}(0)\,\frac{1}{m_\sigma^{*2}}\right)$$

BINDING ENERGY



$CHI/CHIPS_{vac}$



2π PROCESSES AND NUCLEAR SUSCEPTIBILITY

• THE $\pi\pi$ T MATRIX AND THE σ PROPAGATOR

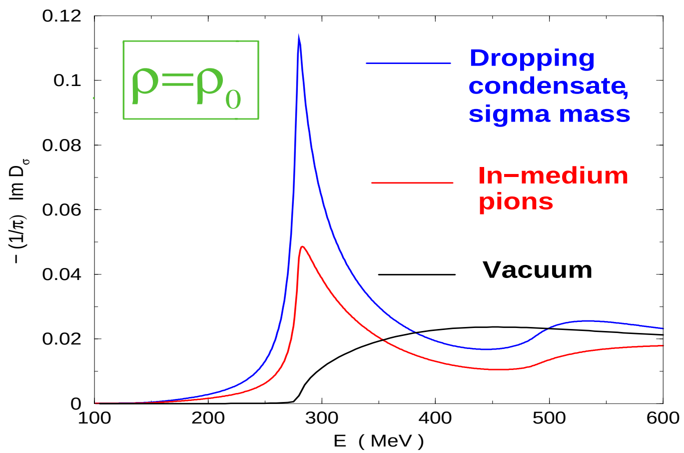
Sigma model ($\pi\pi$ phase shifts) : $T(E) = V(E) + V(E)\,G(E)\,T(E)$

$G(E)$ In-medium two-pion propagator (pion dressed by $p-h$ and $\Delta - h$)

$$G(E) = \int \frac{d\mathbf{q}}{(2\pi)^3} \int \frac{i\,dq_0}{2\pi}\, D_\pi(\mathbf{q}, q_0)\, D_\pi(-\mathbf{q}, E - q_0)$$

$$T(E) = \frac{6\,\lambda(E^2 - m_\pi^2)}{1 - 3\lambda G(E)}\, D_\sigma(E)$$

$$D_\sigma(E) = \left(E^2 - m_\sigma^2 - \frac{6\lambda^2 f_\pi^2 G(E)}{1 - 3\lambda G(E)}\right)^{-1}$$



• PIONIC CONTRIBUTION TO THE NUCLEAR SUSCEPTIBILITY

Vacuum : $(\chi_S)_{vac} = 0.04\ (\chi_{PS})_{vac}$

First order (one p-h insertion) : $\delta\chi_S^{nuclear} = \rho_S\, \chi_S^N = 0.045\ (\chi_{PS})_{vac}$

Full calculation : $\delta\chi_S^{nuclear} = 0.11\ (\chi_{PS})_{vac}$

THE σ (CHIRAL PARTNER OF THE π) vs THE s (CHIRAL INVARIANT)

$$T(E) = \frac{6\,\lambda(E^2 - m_\pi^2)}{1 - 3\lambda G(E)} D_\sigma(E)$$

$$D_\sigma(E) = \left(E^2 - m_\sigma^2 - \frac{6\lambda^2 f_\pi^2 G(E)}{1 - 3\lambda G(E)}\right)^{-1}$$

Strong $\sigma\pi\pi$ coupling

$$T(E) = \frac{6\,\lambda(E^2 - m_\pi^2)}{1 + \frac{3}{2f_\pi^2}(E^2 - m_\pi^2)\,G(E)} D_s(E)$$

$$D_s(E) = \left(E^2 - m_\sigma^2 - \frac{3}{2f_\pi^2}\frac{(E^2 - m_\pi^2)^2\,G(E)}{1 + \frac{3}{2}\frac{E^2 - m_\pi^2}{f_\pi^2}G(E)}\right)^{-1}$$

Weak derivative $s\pi\pi$ coupling

$$D_\sigma(E) = D_s(E) + \frac{3}{2f_\pi^2}\left(1 - 2\,\frac{E^2 - m_\pi^2}{E^2 - m_\sigma^2}\right)\tilde{G}$$

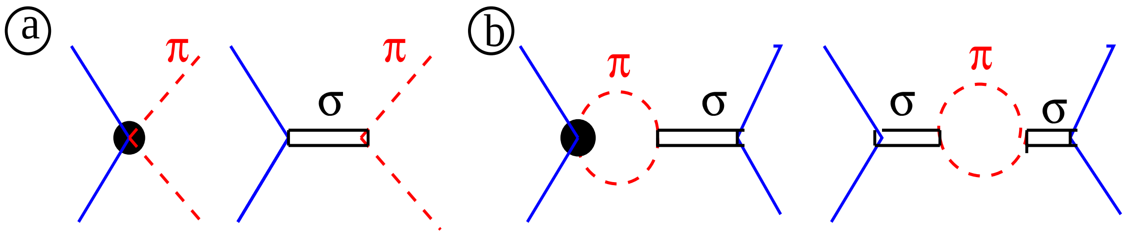
$\tilde{G}$ is the full two-pion propagator : $G = G + \frac{1}{2} G\,V\,\tilde{G}$

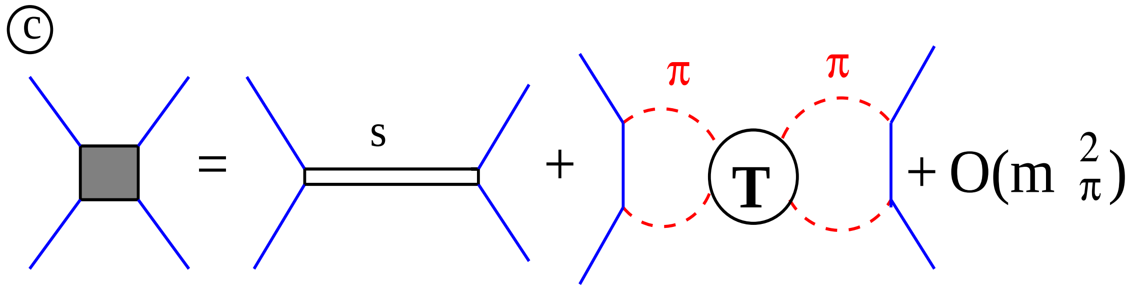
- The chiral invariant s mode is the exchanged meson in the scalar NN potential and is free of many-body effect : weak coupling to in-medium modified 2π states

- The medium effects in the σ propagator and in two-pion processes come from the second 2π modes

THE NN SCALAR POTENTIAL IN THE LINEAR σ FORMULATION

Nucleons exchange not only σ but also 2π states with delicate compensations



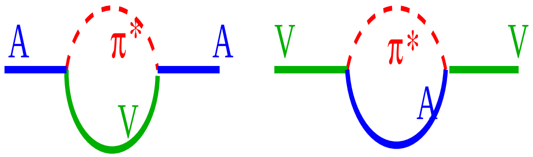


- THE NONLINEAR σ FORMULATION IS MUCH MORE ECONOMICAL
- THE MEDIUM EFFECT IN THE 2π PROCESSES ARE MOSTLY ABSENT IN THE NN SCALAR EXCHANGE

CORRELATOR MIXING

- **AXIAL VECTOR MIXING**

At finite temperature



$$\Pi_V^{\mu\nu}(q;T) = (1-\epsilon)\,\Pi_V^{\mu\nu}(q;T=0) + \epsilon\,\Pi_A^{\mu\nu}(q;T=0)$$

$$\Pi_A^{\mu\nu}(q;T) = (1-\epsilon)\,\Pi_A^{\mu\nu}(q;T=0) + \epsilon\,\Pi_V^{\mu\nu}(q;T=0)$$

$$\epsilon = \frac{T^2}{6\,f_\pi^2} = \frac{2}{f_\pi^2}\int \frac{d\mathbf{k}}{(2\pi)^3}\,\frac{n(\omega_k)}{\omega_k} = \frac{2}{3}\,\frac{<<\Phi^2>>}{f_\pi^2}$$

- **SCALAR-PSEUDOSCALAR MIXING**

$$D_\sigma(E) = D_s(E) + \frac{3}{2f_\pi^2}\left(1 - 2\,\frac{E^2-m_\pi^2}{E^2-m_\sigma^2}\right)\,G$$

$$D_\sigma(E,T=0) \simeq D_s(E) \simeq 1/E^2 - m_\sigma^2$$

For soft thermal pions, $q << E$:

$$3\,G(E,T) = 3\int\frac{d\mathbf{q}}{(2\pi)^3}\int\frac{i\,dq_0}{2\pi}\,D_\pi(\mathbf{q},q_0)\,D_\pi(-\mathbf{q},\,E-q_0) \simeq \frac{<<\Phi^2>>}{E^2-m_\pi^2}$$

$$D_\sigma(E,T) = \left(1 - \frac{<<\Phi^2>>}{f_\pi^2}\right)D_\sigma(E,T=0) + \frac{<<\Phi^2>>}{2\,f_\pi^2}\,D_\pi(E,T=0)$$

CONCLUSION

- Sizeable effects in the nuclear scalar susceptibility

- Scalar and pseudoscalar susceptibilities becomes closer $\longrightarrow$ partial chiral restoration

- Nucleon structure effect compensates the chiral dropping of the sigma mass ; related to the p-h piece of the scalar susceptibility

- $\sigma - 2\pi$ medium effects in two-pion processes related to the pionic nuclear scalar susceptibility

- This effect is not present in the chiral invariant scalar exchange in the NN interaction