

# *High $P_T$ hadron-hadron correlations in Vacuum and in Matter*

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in collaboration with  
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LBNL

Based on,

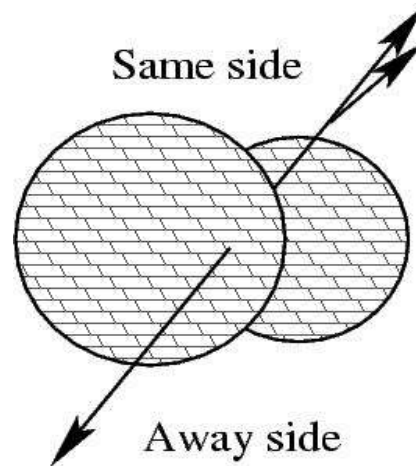
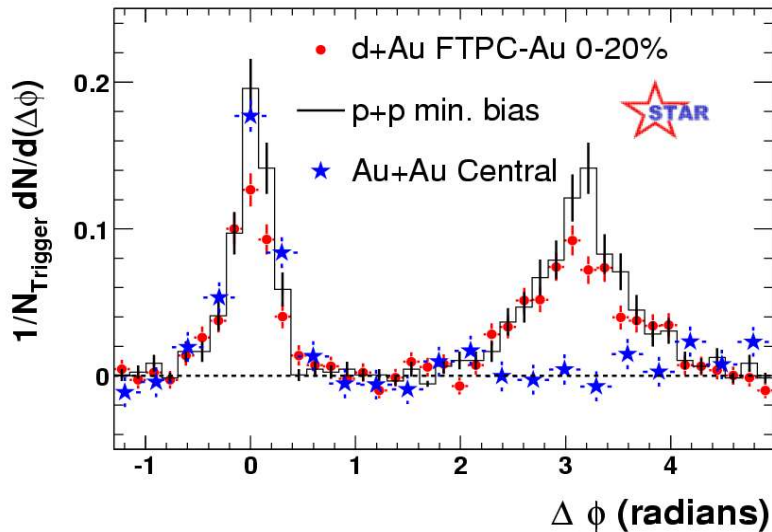
**A. Majumder and Xin-Nian Wang, PRD D. 70, 014007 (2004).**

**A. Majumder, J. Phys. G. 30, S1305 (2004). *Proc. QM2004.***

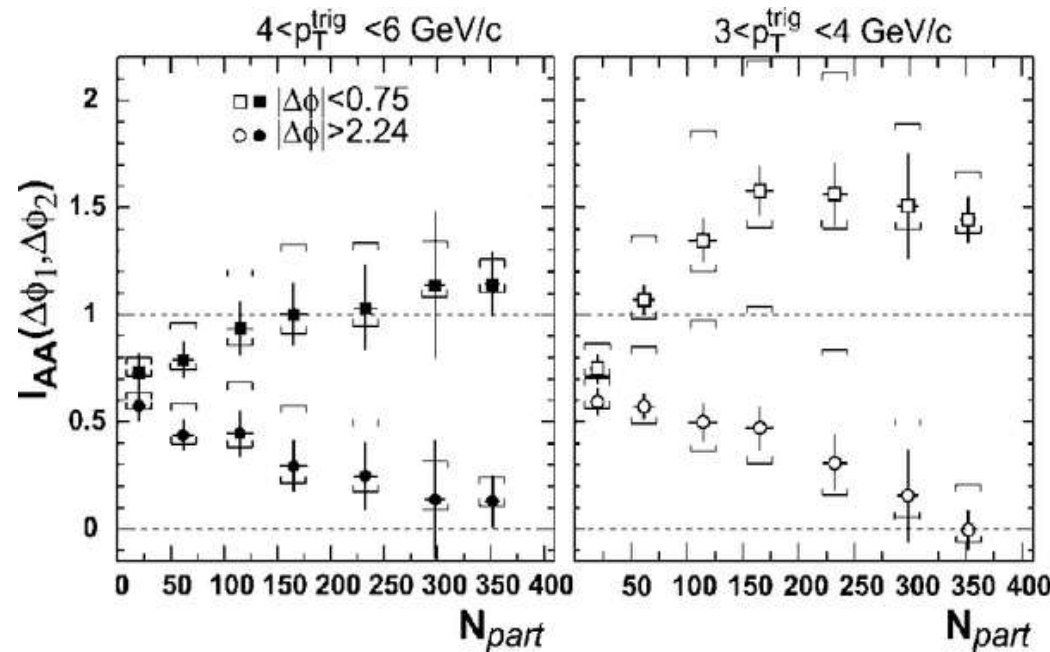
**A. Majumder and Xin-Nian Wang, LBNL-56199, *to be published,***

**A. Majumder, and Xin-Nian Wang J. Phys. G., *Proc. Hot Quarks 2004, hep-ph/0410...***

# Dihadron measurements in heavy-ion coll.



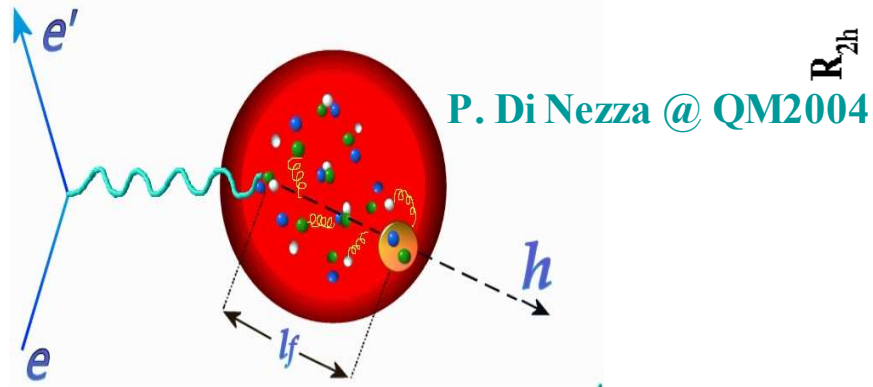
- *d+Au central very similar to p+p*
- *Away side, central Au+Au suppressed*



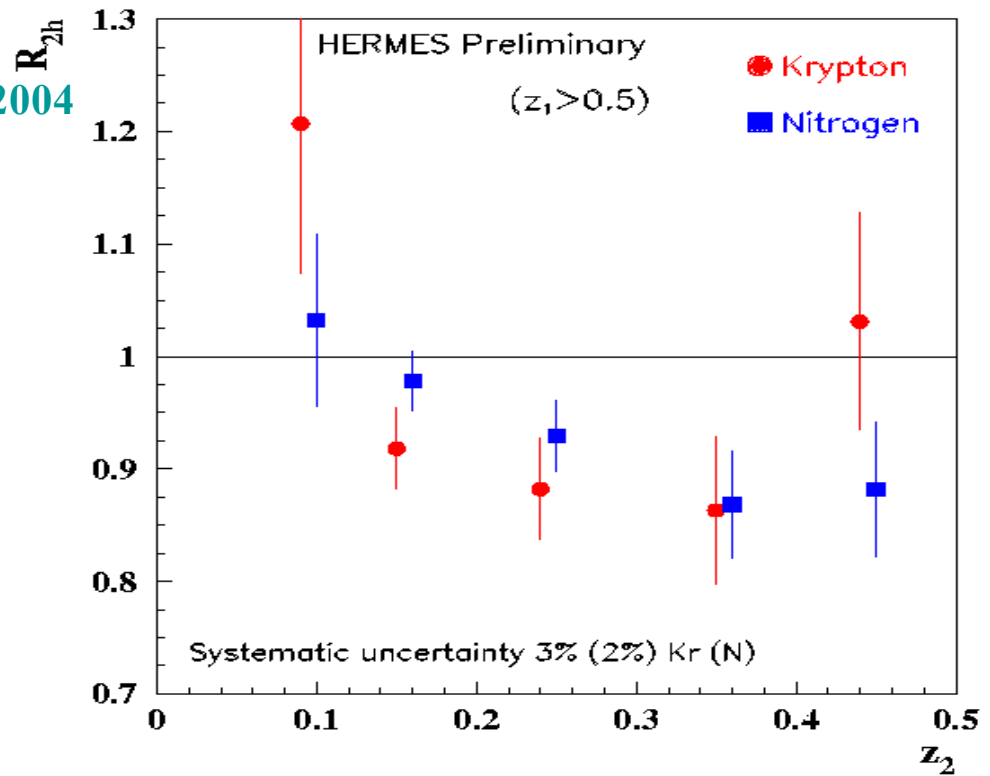
- *Near side, central Au+Au unchanged*

*Some variation with centrality and flavour of trigger*

# RESULTS FROM HERMES, DIS on cold nuclei



- *Always measure a ratio of double to single production*
- *Divide by same ratio in deuterium to remove detector systematics*



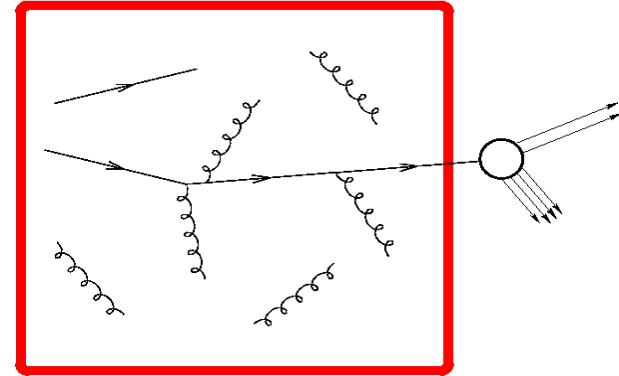
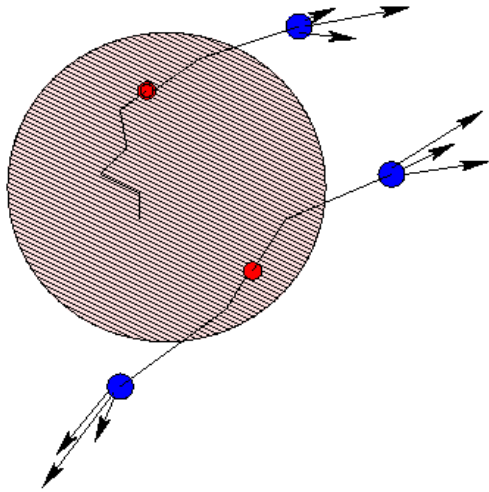
$$R_{2h} = \frac{\text{No. of events with at least 2 hadrons with } z_1 > 0.5}{\text{No. of events with at least one hadron with } z > 0.5}$$

*same ratio on deuterium*

# TWO POSSIBILITIES !

## PARTONIC ENERGY LOSS :

- *High energy partons are created over the entire collision zone*



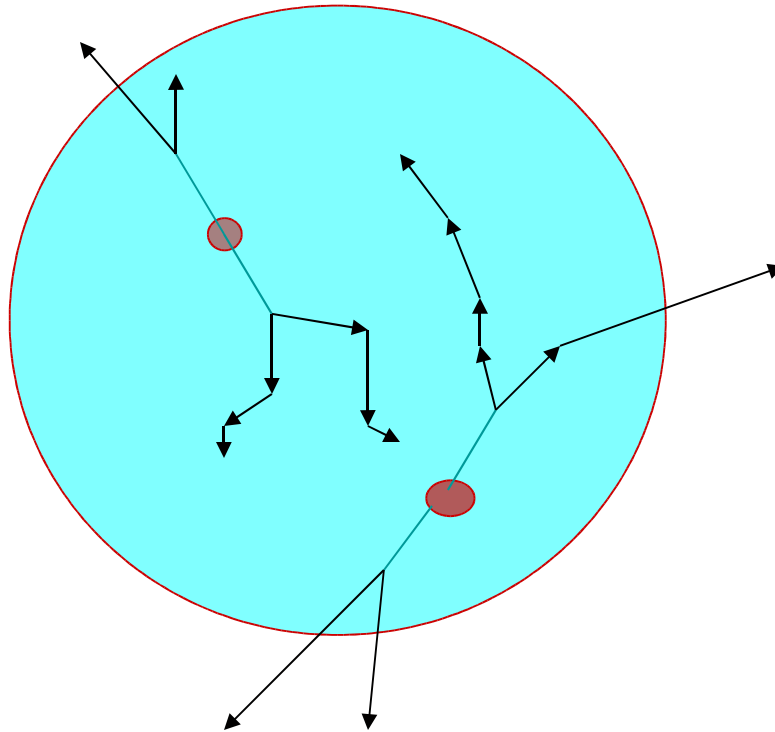
- *Lose energy by partonic interaction, medium may be hadronic or partonic*
- *Emerge as partons and then fragment*
- *Require knowledge of single and double fragmentation functions*

11/12/04  $D_{q,g}^h(z)$

$$D_{q,g}^{h_1 h_2}(z_1, z_2)$$

## *HADRONIC ENERGY LOSS:*

- *Fragmentation occurs inside the hot medium*
- *Hadrons become independent due to scattering*
- *Each hadron suffers the same Energy Loss on average*



*Hadronic scattering models can explain mean single supp. !*  
*C. Greiner et. al. @QM2004, V. Koch (unpublished!)*

- ***NOTE! IN THIS CASE DONT NEED A  $D(z_1, z_2)$***
- ***Probability of observing two hadrons factorizes  $P(1,2) = P(1)P(2)$***
- ***Each probability is suppressed compared to  $p+p$ :  $P(h) = s p(h)$***
- ***Thus the conditional probability is also suppressed compared to  $p+p$  collisions***

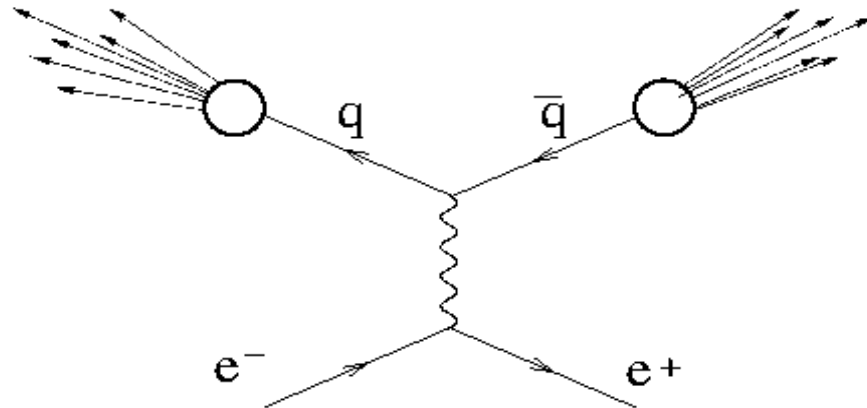
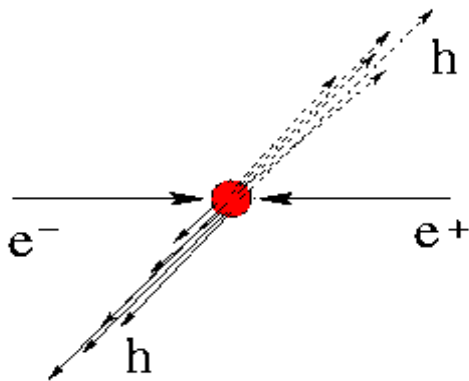
$$\frac{P(1,2)}{P(1)} = \frac{P(1)P(2)}{P(1)} = P(2) = sp(2) = s \frac{p(1)p(2)}{p(1)}$$

***$P(h)$  is for  $A+A$ ;       $p(h)$  is for  $p+p$***

- ***Hadronic absorption models cannot explain the double inclusive spectrum***

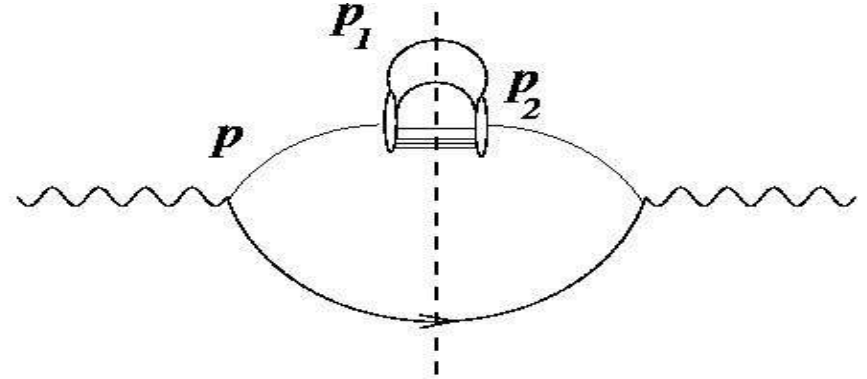
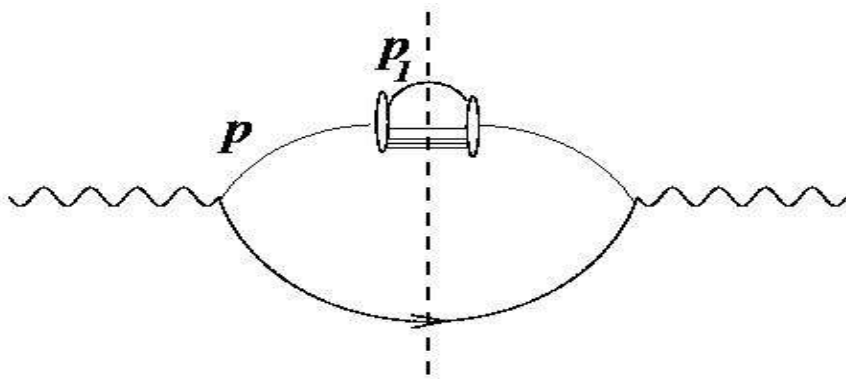
# Defining the Dihadron fragmentation function!

- *Fragmentation functions have to be universal*
- *We need a definition in terms of operators*
- *Start with simple system :  $e^+ e^-$ , is factorization valid*
- *Derive evolution (vacuum splitting functions)*
- *Measure at  $\mu$  and predict its evolution to scale  $Q$*



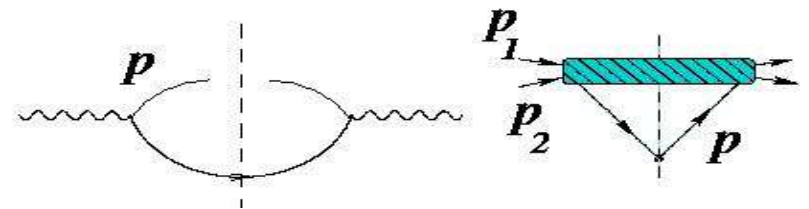
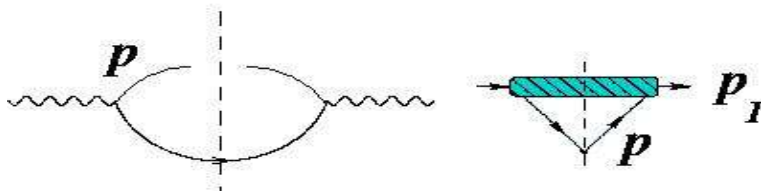
$$W^{\mu\nu} = \sum_{S-1} e_q^2 \int \frac{d^3 p_1 d^3 k}{4 E_1 E_k (2\pi)^6} (2\pi)^4 \delta(q - p_1 - k - \sum_{S-1} p_f)$$

$$\langle 0 | \bar{\psi} \gamma^\mu \psi | k p_1 S-1 \rangle \langle k p_1 S-1 | \bar{\psi} \gamma^\nu \psi | 0 \rangle$$

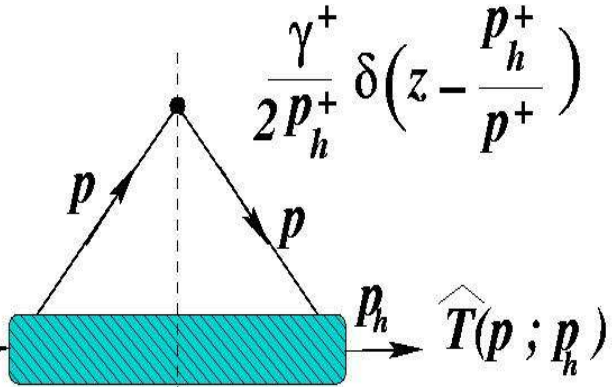


**Collinear approximation:**  $p_h^+, p^+, p_1^+, p_2^+ \gg p_h^-, p^-, p_1^-, p_2^-, p_{perp}$

**Can factorize matrix elements from hard part:**  $Tr[\gamma^\alpha p_\alpha \gamma^\mu \gamma^\beta k_\beta \gamma^\nu] Tr[\frac{\gamma^+}{2 p^+} \hat{T}]$



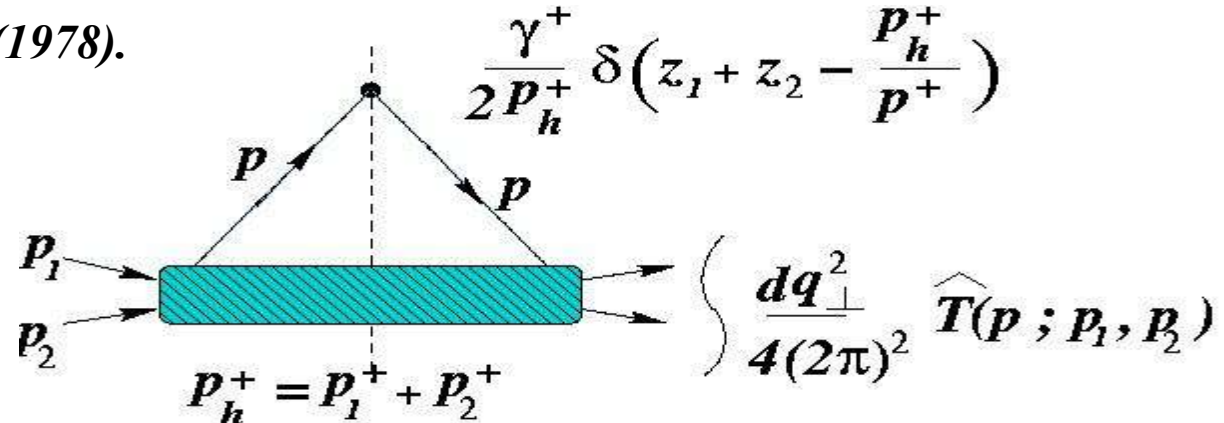
**Use of universal single and double fragmentation functions defined in  $e^+e^-$**

$$D_q(z) = \frac{z^3}{2} \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left[ \frac{\gamma^+}{2p_h^+} \delta\left(z - \frac{p_h^+}{p^+}\right) \hat{T}_q(p, p_h) \right] = \frac{z^3}{2} \times p_h \times \hat{T}(p; p_h)$$


where  $\hat{T}_q(p, p_h) = \int d^4 x e^{-ip \cdot x} \sum_{S-1} \langle 0 | \psi(0) | p_h, S-1 \rangle \langle p_h, S-1 | \bar{\psi}(x) | 0 \rangle$

$$D_q(z_1, z_2) = \int \frac{dq_{\perp}^2}{8(2\pi)^2} \frac{z^4}{4z_1 z_2} \int \frac{d^4 p}{(2\pi)^4} \text{Tr} \left[ \frac{\gamma^+}{2p_h^+} \delta\left(z_1 + z_2 - \frac{p_h^+}{p^+}\right) \hat{T}_q(p, p_1, p_2) \right]$$

**A. Mueller, PRD 18, 3705 (1978).**

$$= \frac{z^4}{z_1 z_2} \left( \frac{dq_{\perp}^2}{4(2\pi)^2} \hat{T}(p; p_1, p_2) \right)$$


# *NLO contribution from quark dihadron fragmentation function*

$$\begin{aligned}
 & \text{Diagram 1} = \text{Diagram 2} \otimes \text{Diagram 3} \\
 & \text{Diagram 2} = \text{Diagram 4} \otimes \text{Diagram 5} \otimes \text{Diagram 6} \\
 & \text{Diagram 4} = \left| \text{Diagram 7} \right|_2 \otimes \left| \text{Diagram 8} \right|_2 \otimes \left| \text{Diagram 9} \right|_2 \\
 & = \sigma_0 \otimes P_{q \rightarrow qg}(y) \otimes D_q \left( \frac{z_1}{y}, \frac{z_2}{y} \right)
 \end{aligned}$$

Diagram 1: A quark line with momentum  $p$  and a dihadron fragmentation function (DF) insertion. The DF is shown as a shaded region with momenta  $p_1$  and  $p_2$  entering and  $p$  exiting.

Diagram 2: A convolution of a hard scattering cross-section (a quark line with a loop) and a DF.

Diagram 3: A hard scattering cross-section with a quark line and a loop. The loop is labeled with momentum  $l$ . The cross-section is multiplied by a delta function:  $\frac{\gamma^+}{2P_h^+} \delta(z_1 + z_2 - \frac{P_h^+}{P^+})$ .

Diagram 4: A convolution of a hard scattering cross-section and a DF.

Diagram 5: A hard scattering cross-section with a quark line and a loop. The loop is labeled with momentum  $l$ . The cross-section is multiplied by a delta function:  $\frac{\gamma^+}{2P_h^+} \delta(z_1 + z_2 - \frac{P_h^+}{P^+})$ .

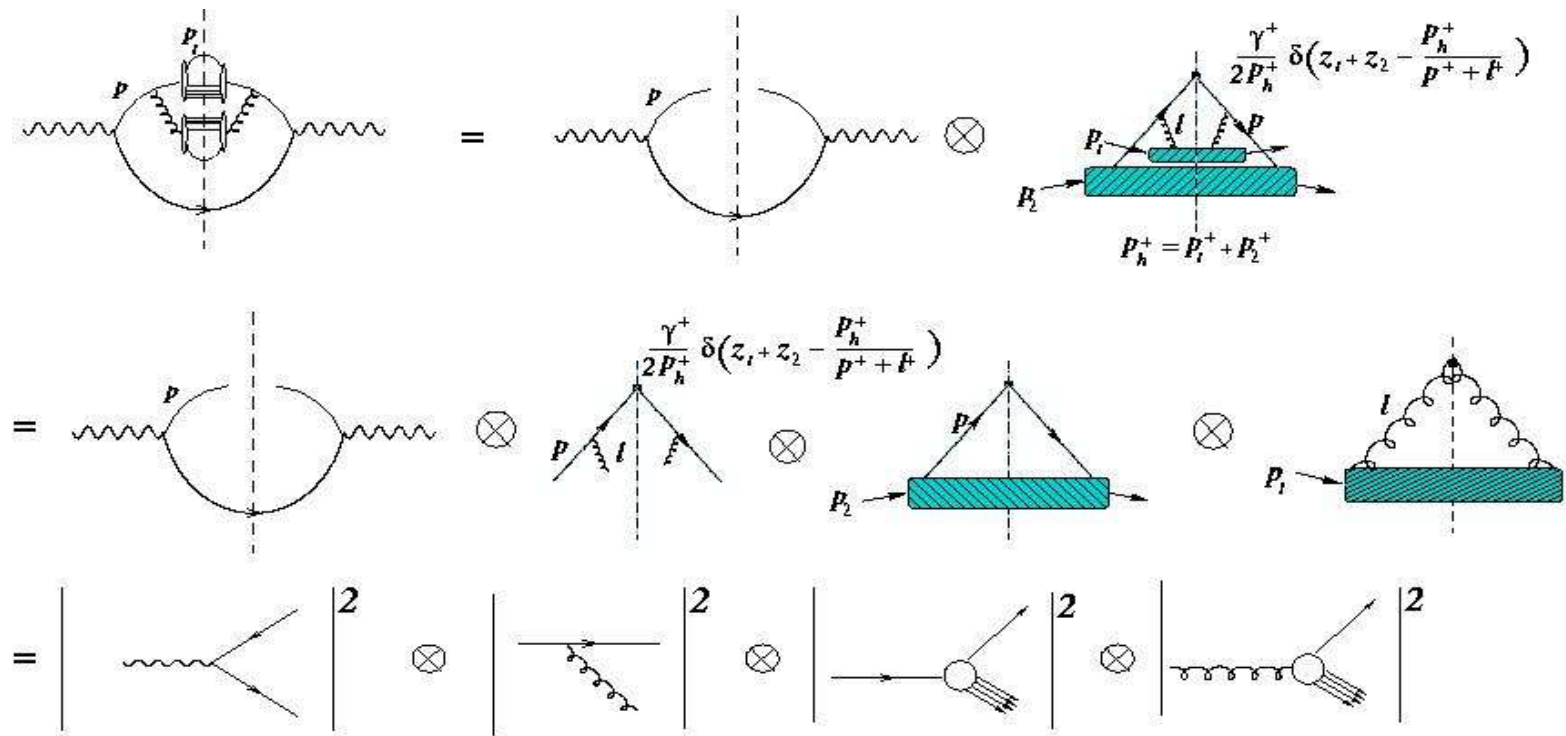
Diagram 6: A convolution of a hard scattering cross-section and a DF.

Diagram 7: A hard scattering cross-section with a quark line and a loop. The loop is labeled with momentum  $l$ . The cross-section is multiplied by a delta function:  $\frac{\gamma^+}{2P_h^+} \delta(z_1 + z_2 - \frac{P_h^+}{P^+})$ .

Diagram 8: A hard scattering cross-section with a quark line and a loop. The loop is labeled with momentum  $l$ . The cross-section is multiplied by a delta function:  $\frac{\gamma^+}{2P_h^+} \delta(z_1 + z_2 - \frac{P_h^+}{P^+})$ .

Diagram 9: A hard scattering cross-section with a quark line and a loop. The loop is labeled with momentum  $l$ . The cross-section is multiplied by a delta function:  $\frac{\gamma^+}{2P_h^+} \delta(z_1 + z_2 - \frac{P_h^+}{P^+})$ .

# NLO contribution from independent quark and gluon fragmentation



$$= \sigma_0 \otimes \hat{P}_{q \rightarrow qg}(y) \otimes D_q\left(\frac{z_1}{y}\right) \otimes D_g\left(\frac{z_2}{1-y}\right)$$

*The hat indicates there is no virtual correction.*  
*Also no infra-red divergence as hadrons from both partons detected*  
*However, perturbative corrections under control only if  $\mu_{\perp}^2 \gg \lambda_{QCD}^2$*

# *Dihadron fragmentation in $e^+ e^-$ Collisions*

*The basic process may be factorized as:*

$$\frac{d^2 \sigma}{d z_1 d z_2} = \sigma_0 [D_q(z_1, z_2, \mu) + D_{\bar{q}}(z_1, z_2, \mu)]$$

$\sigma_0 =$  *Hard Cross section*

$D_q(z_1, z_2, \mu) =$  *Dihadron fragmentation function*

*Can be factorized from hard process if*  $\lambda_{QCD}^2 \ll \mu^2 \ll Q^2$

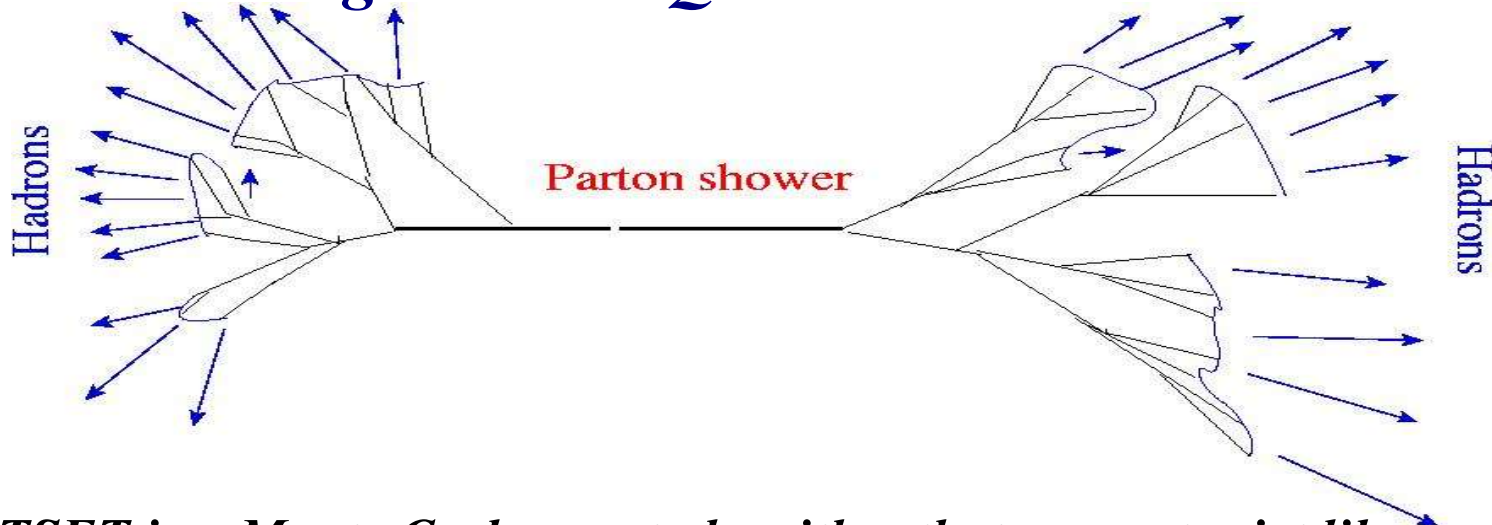
*Measure the function at the scale  $\mu$ , can be done in 2 ways*

*Factorized Distribution:  $D(z_1, z_2, \mu) = D(z_1, \mu) D(z_2, \mu)$*

*Event generator distribution:  $D(z_1, z_2, \mu) = \frac{1}{N_{events}} \frac{dN}{dz_1 dz_2}$*

## *Evolution of full quarks and gluons*

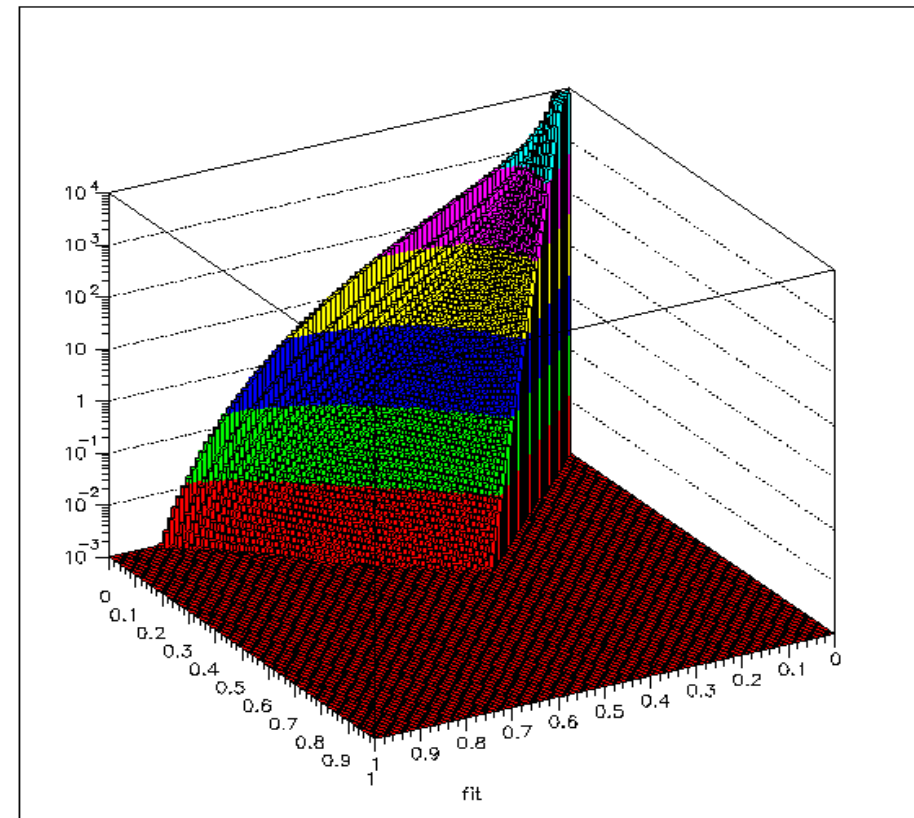
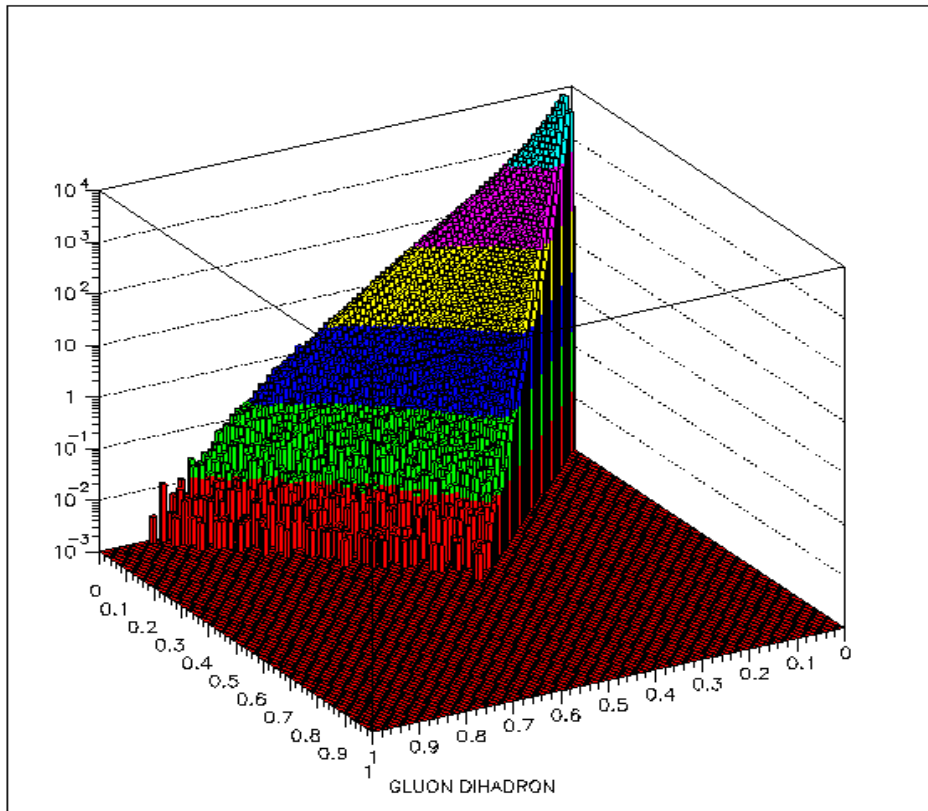
- *No experimental data to date on dihadron frag. In  $e^+e^-$*
- *Phenomenological event generators can explain most data!*
- *Most successful of these is JETSET*
- *Use a tuned JETSET to measure dihadrons at scale  $\mu$*
- *Measure at higher scale  $Q$  and check with derived DGLAP*



*JETSET is a Monte Carlo event algorithm that generates jet like events with a parton shower followed by a string fragmentation routine to get hadrons. It has many parameters tuned to fit almost all experimental data.*

# Results from Event generators:

*a bit ragged (Monte Carlo), fit a function to it !*



$$D(z_1, z_2) = N z_1^{\alpha_1} z_2^{\alpha_2} (z_1 + z_2)^{\alpha_3} (1 - z_1)^{\beta_1} (1 - z_2)^{\beta_2} (1 - z_1 - z_2)^{\beta_3}$$

# *Modification in vacuum = DGLAP evolution*

*Non-singlet quarks*  $D_{NS} = D_q - D_{\bar{q}}$

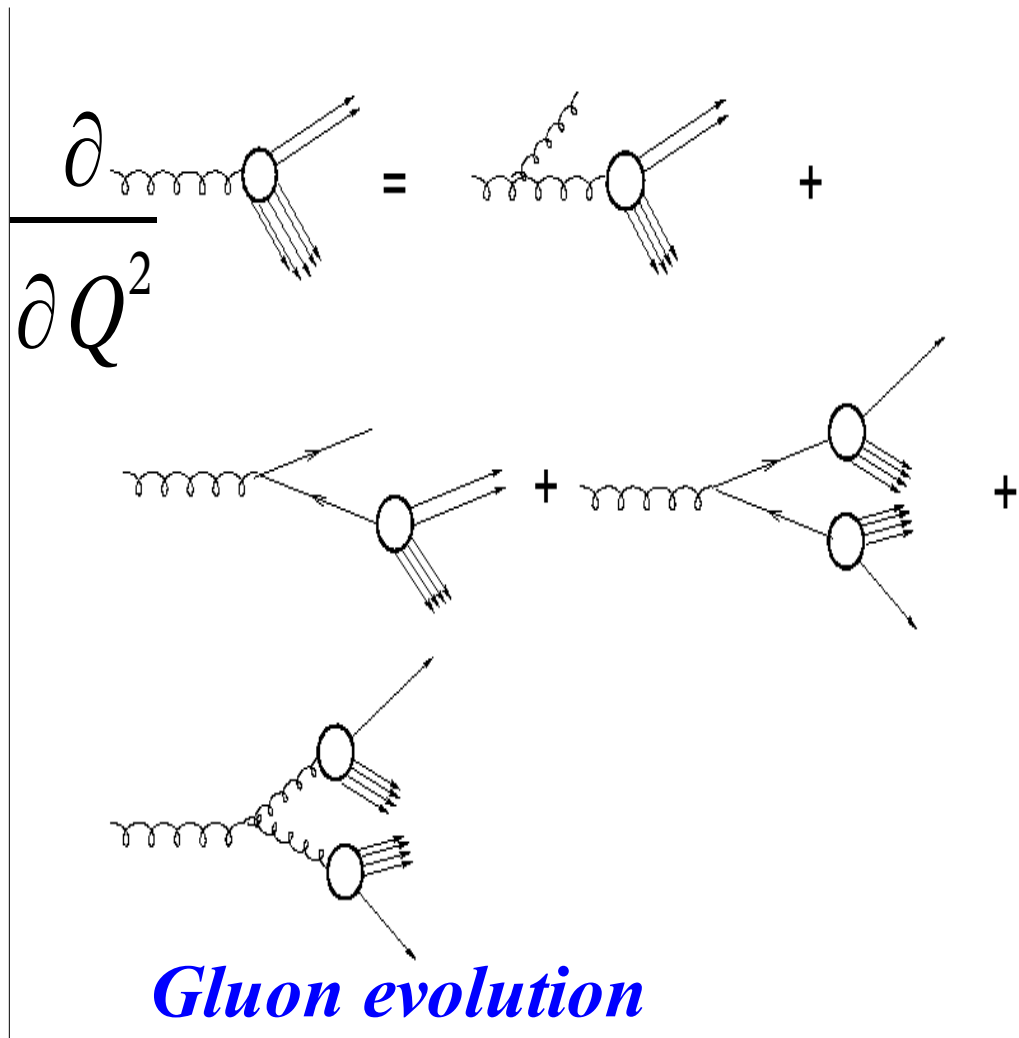
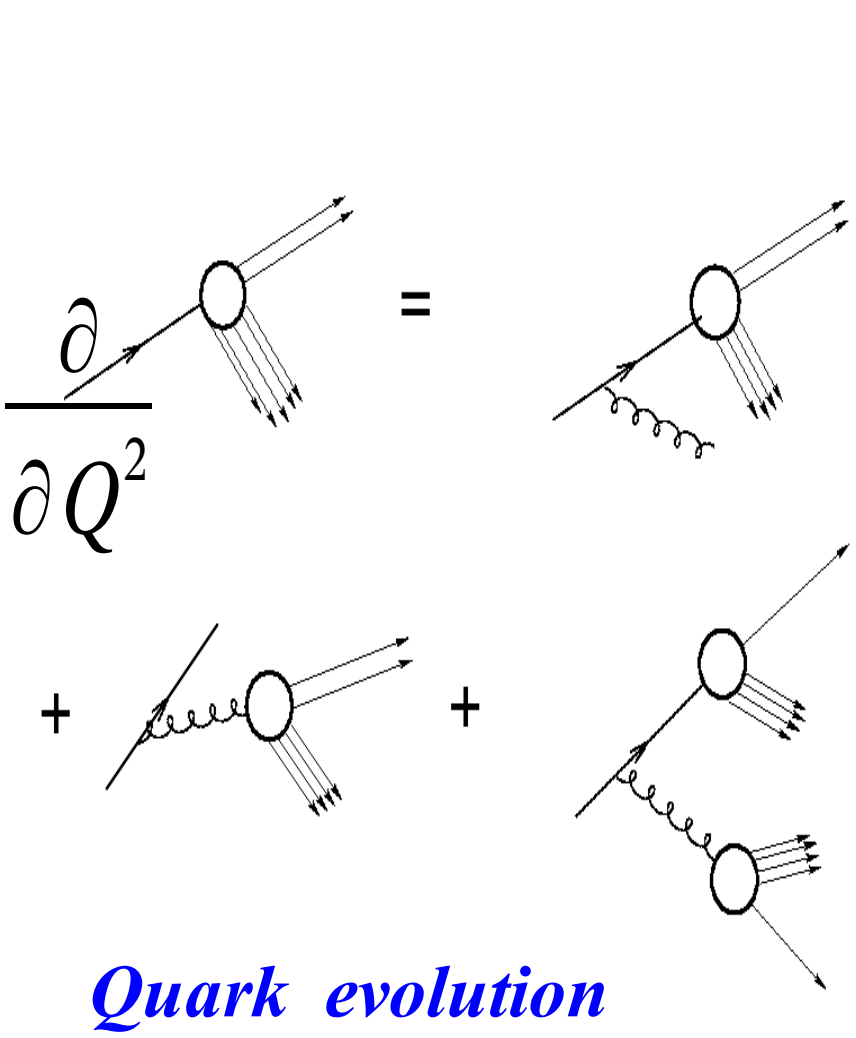
*Simpler: contribution from gluon fragmentation cancels out*

*Single evolution:* 
$$\frac{\partial D_{NS}(z, Q^2)}{\partial \log Q^2} = \int_z^1 \frac{dy}{y} P_{q \rightarrow qg}(y) D_{NS}(z/y, Q^2)$$

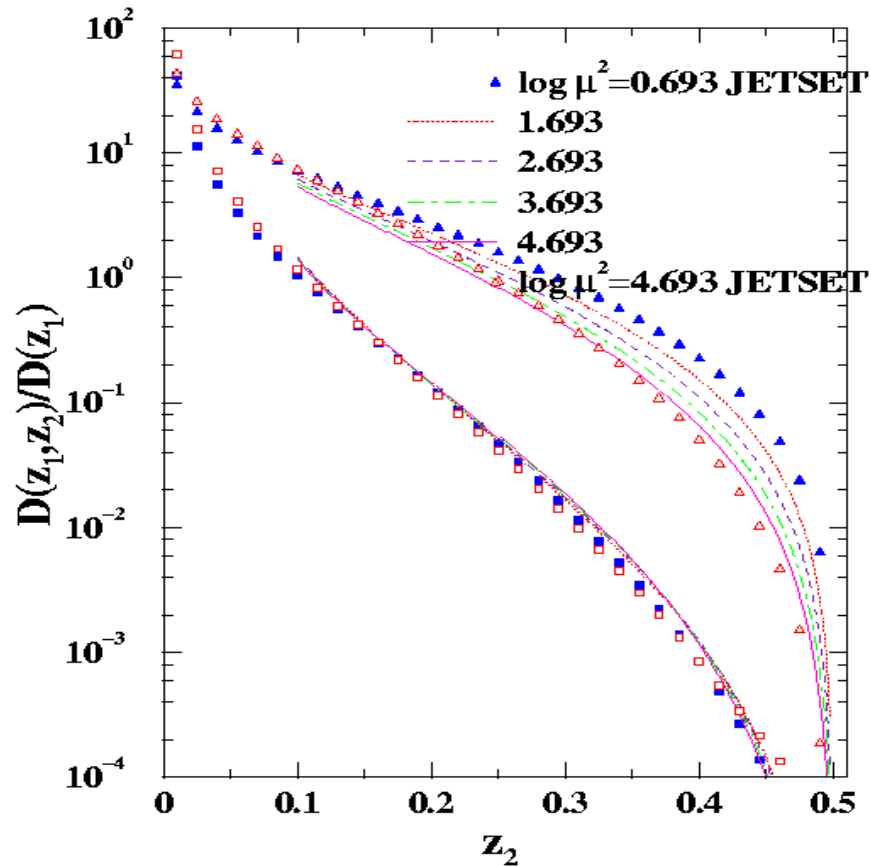
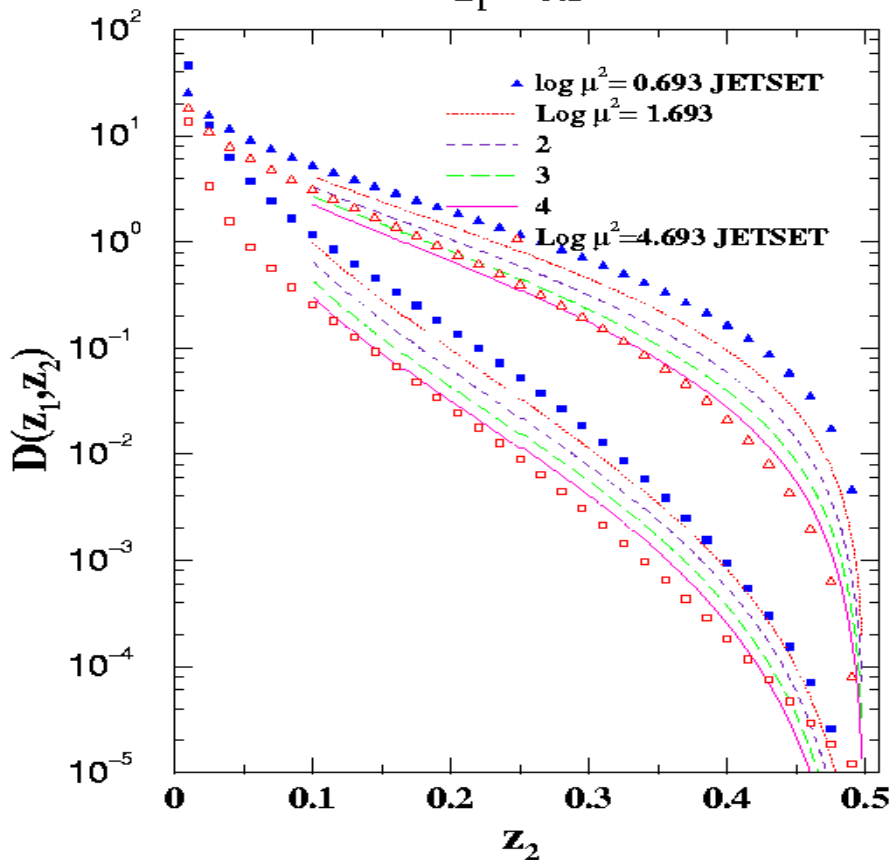
*Double evolution:* 
$$\begin{aligned} \frac{\partial D_{NS}(z_1, z_2, Q^2)}{\partial \log Q^2} &= \int_{z_1, z_2}^1 \frac{dy}{y^2} P_{q \rightarrow qg}(y) D_{NS}(z_1/y, z_2/y, Q^2) \\ &\quad + \int_{z_1}^{1-z_2} \frac{dy}{y(1-y)} \hat{P}_{q \rightarrow qg}(y) D_q(z_1/y, Q^2) D_g(z_2/(1-y), Q^2) \end{aligned}$$

$\hat{P} = P - \text{virtual corrections}$

- *Evolving to a higher scale  $Q$   $\rightarrow$  solving DGLAP equations*
- *Set of coupled differential equations containing the following processes: for quarks and gluons..*



$z_1 = 0.5$



*Quark and Gluon evolution fits event generator data very well!*

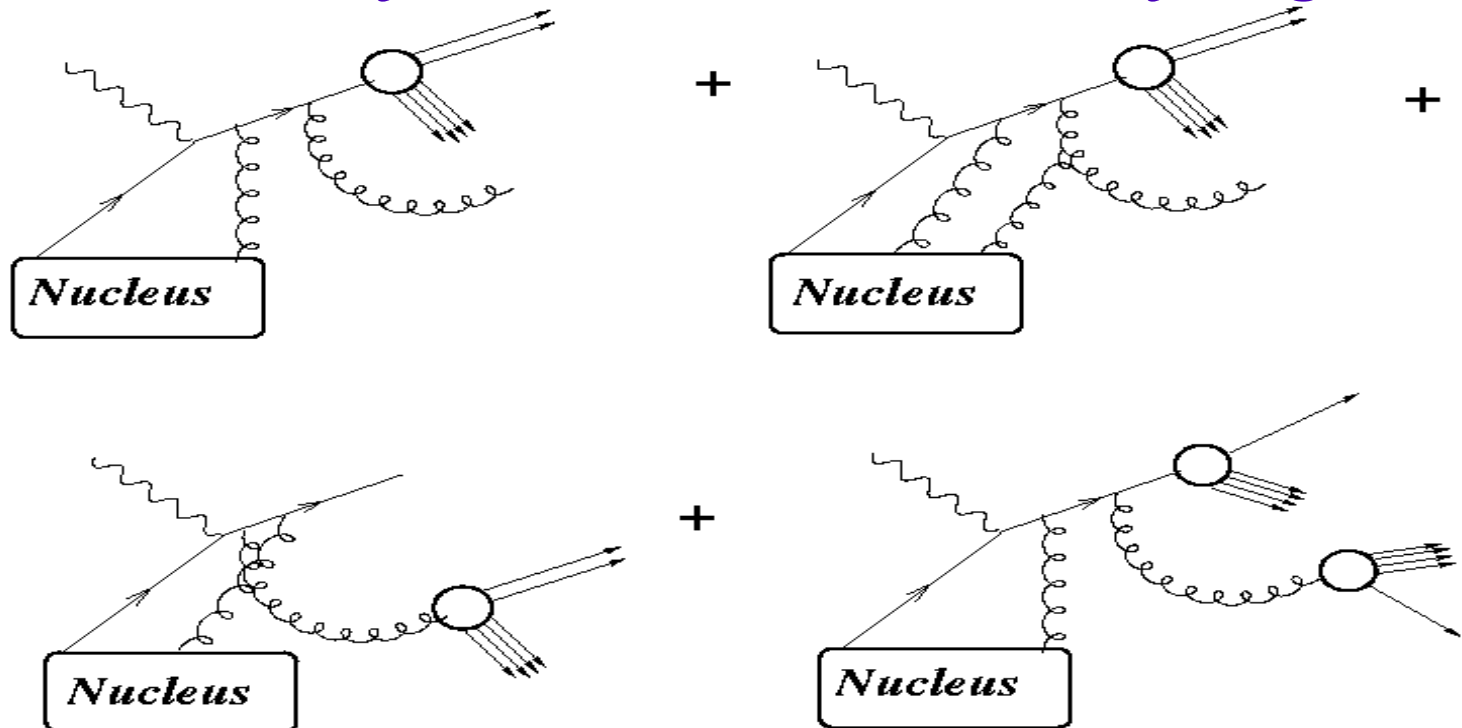
*Thus we can understand evolution of DFF from QCD.*

*Note: the double to single ratio shows little change*

# Medium modification

- Apply to DIS of Nuclei (**HERMES** expt. at **DESY**)
- A parton in a nucleon is struck by EM probe
- Parton scatters in medium and then exits & fragments
- Fragmentation function is medium modified.

The medium modification also has new set of diagrams!

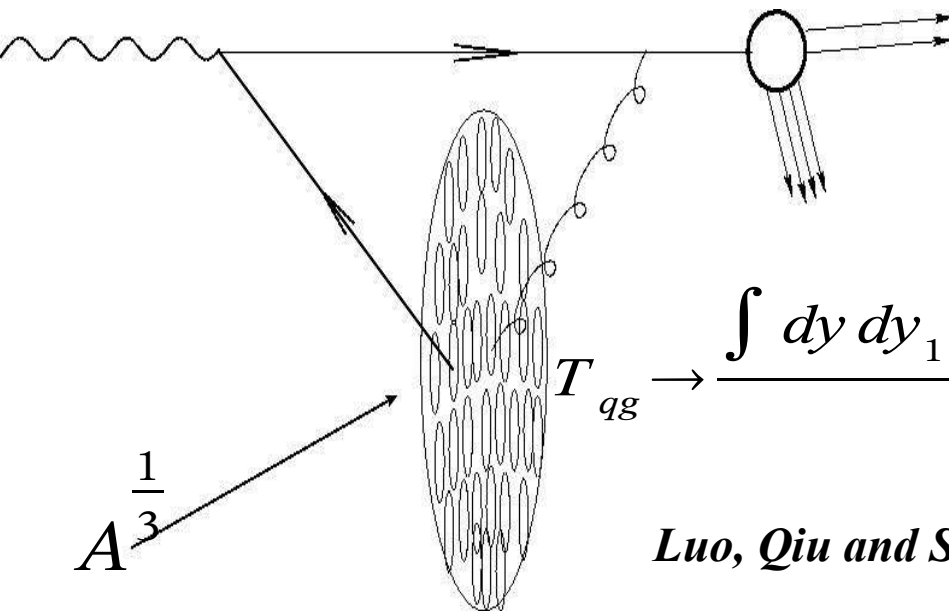


*DIS followed by di hadron fragmentation from a large nuclei may be generally expressed as*

$$\frac{d^2 W^{\mu\nu}}{dz_1 dz_2} = \int dx f_q^A(x) H^{\mu\nu} \tilde{D}^{h_1 h_2}(z_1, z_2)$$

$\tilde{D}$  = medium modified fragmentation function

$$\tilde{D}(z_1, z_2, \mu^2) = D(z_1, z_2, \mu^2) + \frac{\alpha_s}{2\pi} \int_0^{\mu^2} \frac{dl_{\perp}^2}{l_{\perp}^2} \int \frac{dy}{y^2} \left( \frac{1+y^2}{1-y} T_{qg}(x, y, Q^2, l_{\perp}) + V.C. \right) D(z_1/y, z_2/y, \mu^2)$$



$$T_{qg} \rightarrow \frac{\int dy dy_1 dy_2 \langle A | \bar{\psi}(y) F(y_1) F(y_2) \psi(0) | A \rangle}{f^A(x)}$$

*Luo, Qiu and Sterman PRD 50, 1951 (1994).*

# Results for DIS on Nuclei: energy loss in cold matter

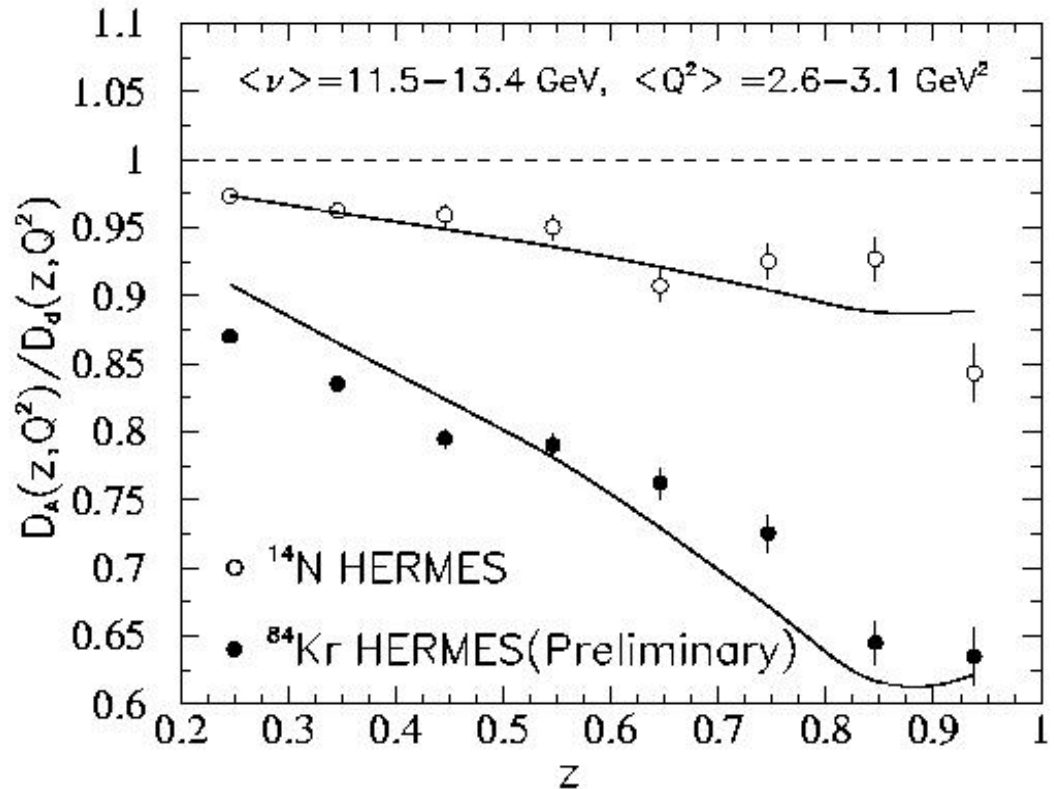
*Single inclusive  
suppression*

*Gaussian distribution  
of nucleons used*

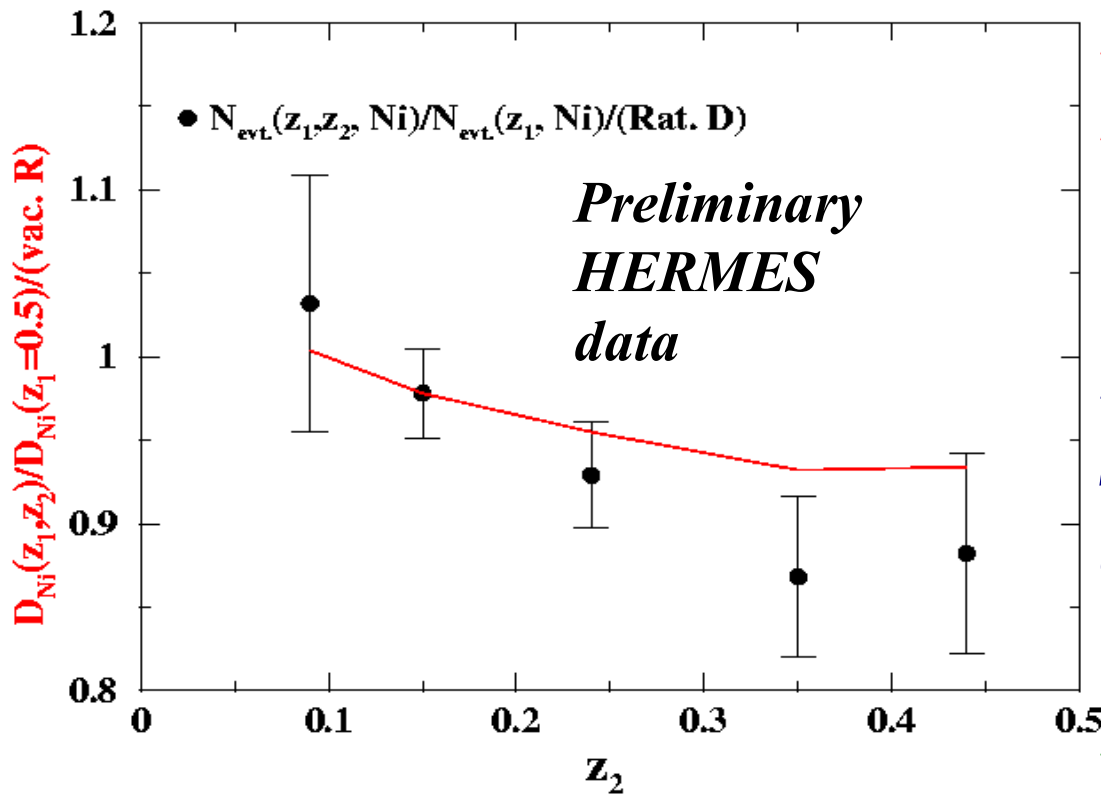
*Very good agreement  
with data*

*Over all normalization  
is now fixed:*

*No free parameters for  
double calculation*



**PRELIMINARY!**



*The theory curve is the number of pairs with one hadron at  $z_1 = 0.5$  and one at  $z_2$ .*

*The expt. curve is the number of events with a subleading hadron at  $z_2$ , and  $z_1 > 0.5$ .*

*Gaussian approx. for nuclear density used!*

*Theory curve:  $(FF(2h)/FF(1h) \text{ in } A) / (FF(2h)/FF(1h) \text{ in vac.})$*

$$\text{Expt ratio} = \frac{\text{No. of events with at least 2 hadrons with } z_1 > 0.5}{\text{No. of events with at least one hadron with } z > 0.5}$$

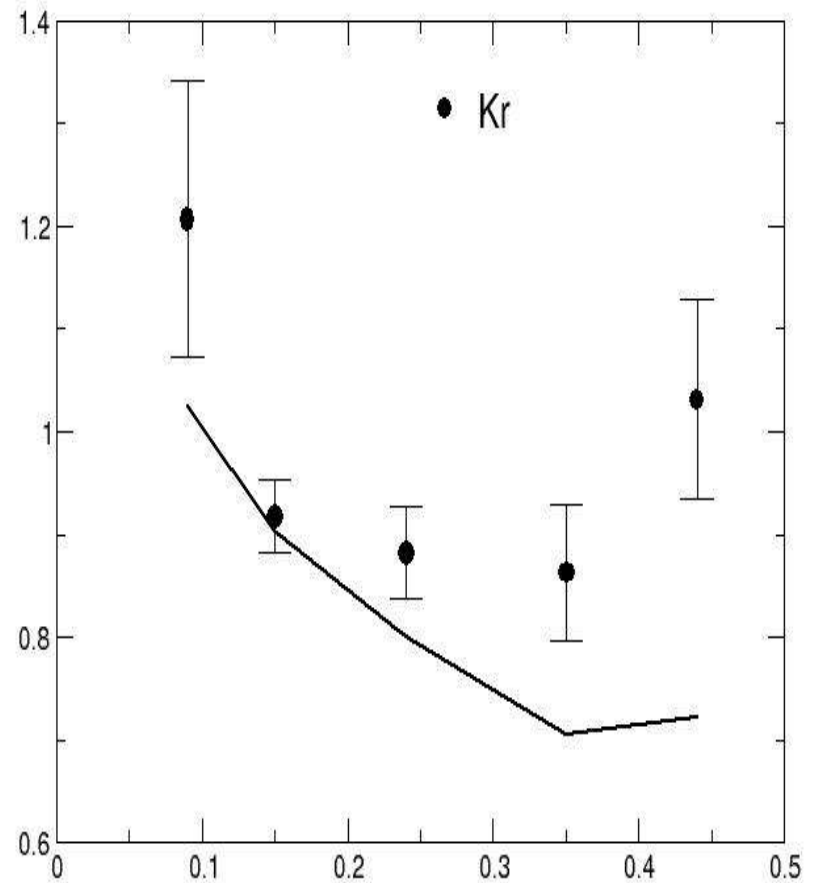
*same ratio on deuterium*

*Gaussian approximation  
not so good for large  
Nuclei like Kr.*

*Hard sphere works better!*

*Remember, no free  
parameters used in plot*

$$dE/dx = 0.5 \text{ GeV/fm}$$



*Perhaps at Larger A one needs to go to higher order power  
corrections.*

# Medium modification in a deconfined medium

$$d\sigma = \int dx_1 f_q^A(x_1) f_q^A(x_2) d\sigma_{hard} \tilde{D}^h(z)$$

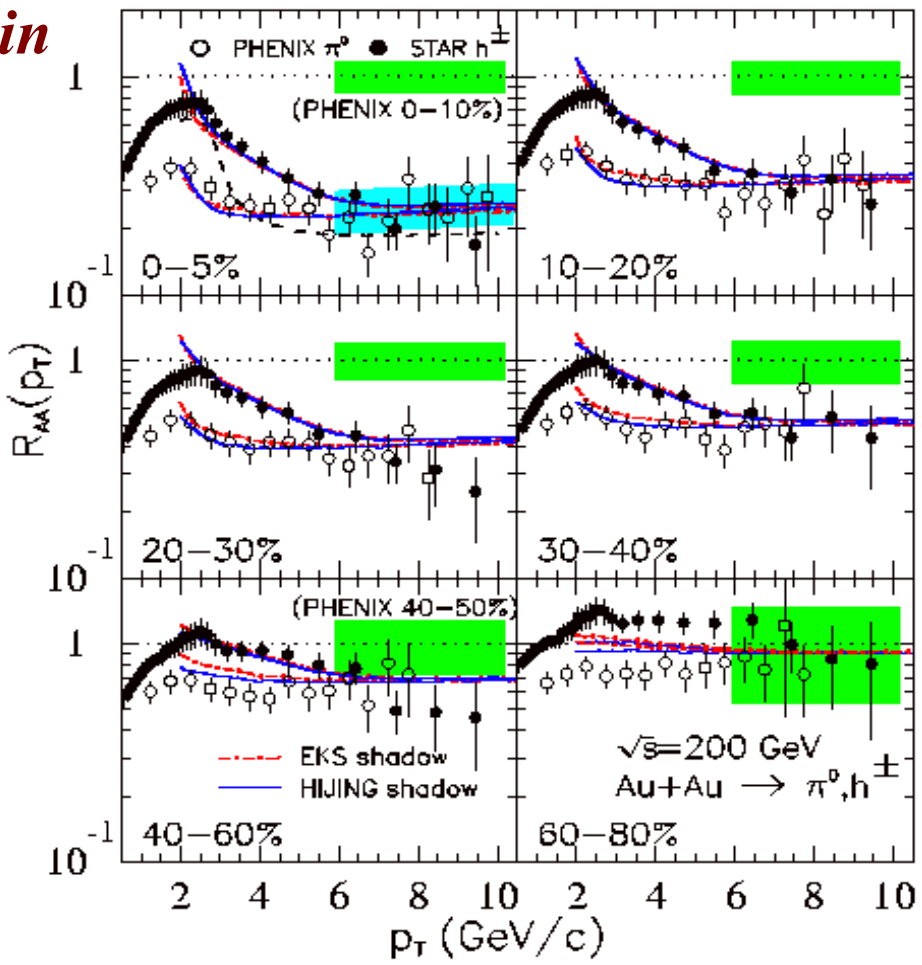
X.N.Wang, \nucl-th 0305010

Calculate medium modification in a 1-D expanding medium,

Dial up gluon density to get suppression.

Gluon density proportional to number of participants

$dE/dx \sim 14 \text{ GeV/fm}$

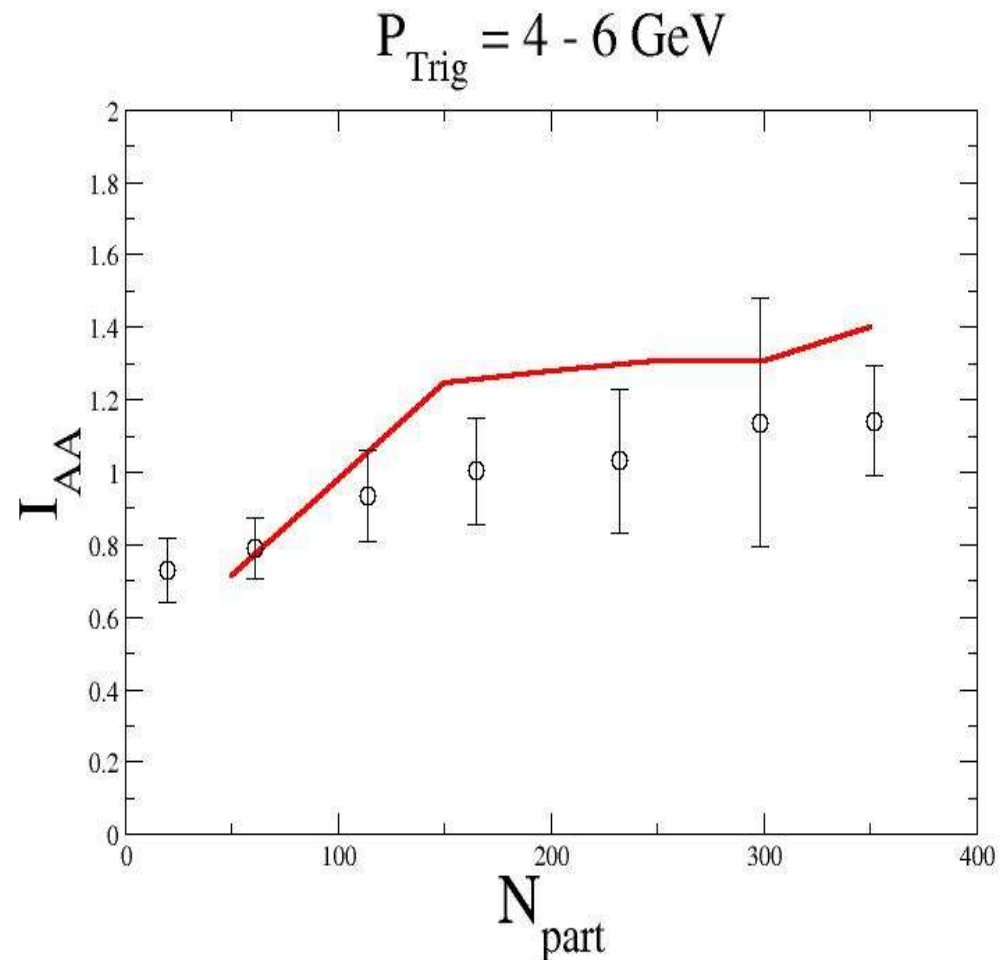


# *Dihadron results for hot medium*

*Very preliminary  
estimate for the same  
side two body  
correlation*

*Results include the  
effect of trigger bias.*

*Initiating parton in  
a heavy-ion collision  
has higher energy  
than that in p-p  
collision..*



# *Summary & Conclusions!*

- *A unified picture of jet quenching in cold and hot matter*
- *Modification of fragmentation functions*
- *Single inclusive and double inclusive measurements*
- *Modification is a partonic effect.*
- *Defined a new object: Dihadron fragmentation function*
- *Medium modification from  $A$  enhanced power corrections  
in **DIS***
- *Extended formalism to modification in hot matter*

Back Up ..

## *Medium modification*

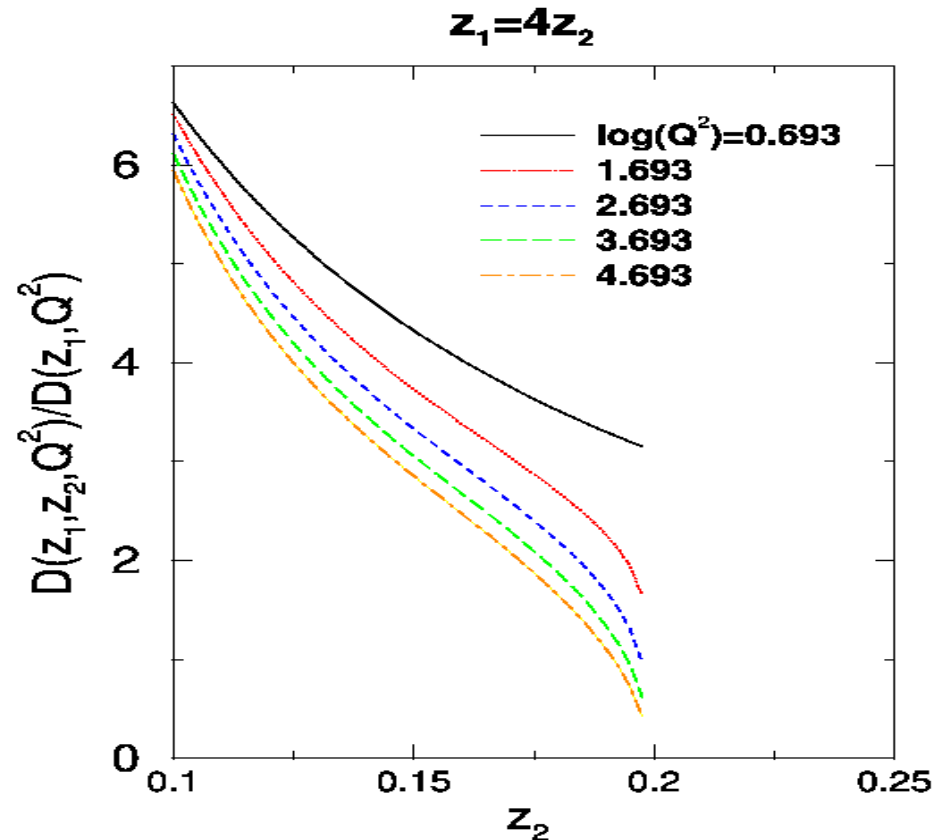
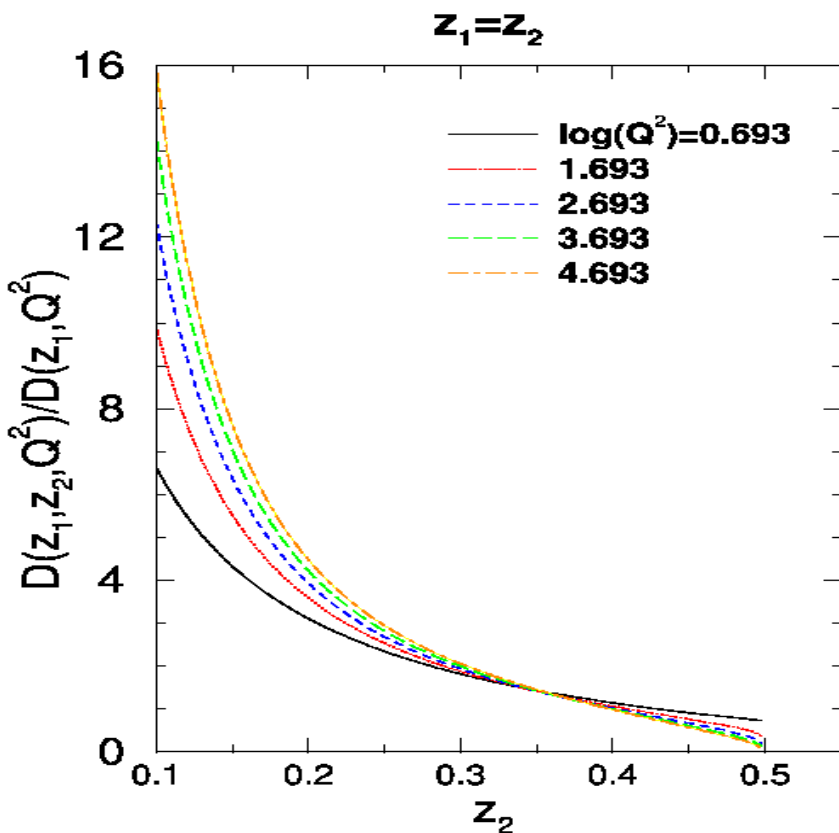
- *Apply to DIS of Nuclei (HERMES expt. at DESY)*
- *A parton in a nucleon is struck by EM probe*
- *Parton traverses cold medium and then fragments*
- *Fragmentation function is medium modified.*

*The medium modification equation looks very similar to the vacuum evolution equation...*

$$\begin{aligned}\tilde{D}_q(z_1, z_2, \mu^2) = & D_q(z_1, z_2, \mu^2) + \int dl_T \int_{z_1+z_2}^1 \frac{dy}{y^2} \tilde{P}_{q \rightarrow qg}(y) D(z_1/y, z_2/y, \mu^2) \\ & + \int dl_T \int_{z_1}^{1-z_2} \frac{dy}{y(1-y)} \hat{\tilde{P}}_{q \rightarrow qg}(y) D(z_1/y, \mu^2) D(z_2/(1-y), \mu^2)\end{aligned}$$

$\tilde{D}$  = *medium modified fragmentation function*

$\tilde{P}$  = *medium modified splitting function*



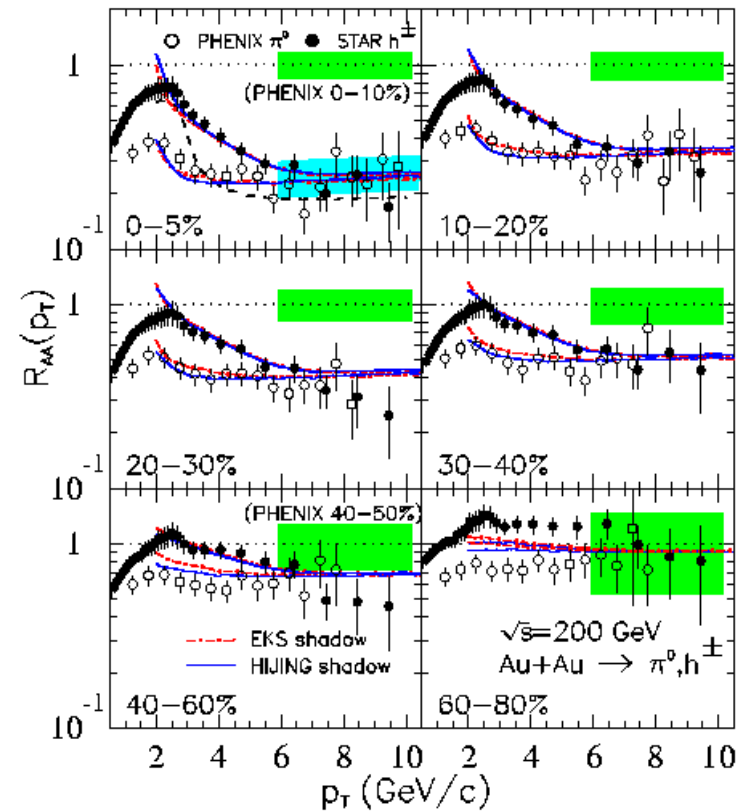
*Measure the function at the scale  $\mu$ , from JETSET*

*JETSET is a Monte Carlo event algorithm that generates jet like events with a parton shower followed by a string fragmentation routine to get hadrons. It has many parameters tuned to fit almost all experimental data.*

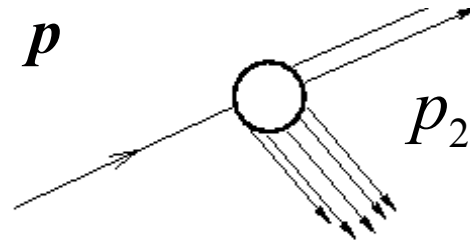
• *Partonic energy loss models explain single inclusive suppression pretty well !*  
*(GLV, BDMPS, WGZ, SW)*

*All models require a high density of scattering centers*  
*High density seen as evidence of QGP.*

*To explain double inclusive spectra requires a new phenomenological object: Dihadron fragmentation function!*



$$D_{q,g}^{h_1 h_2}(z_1, z_2)$$

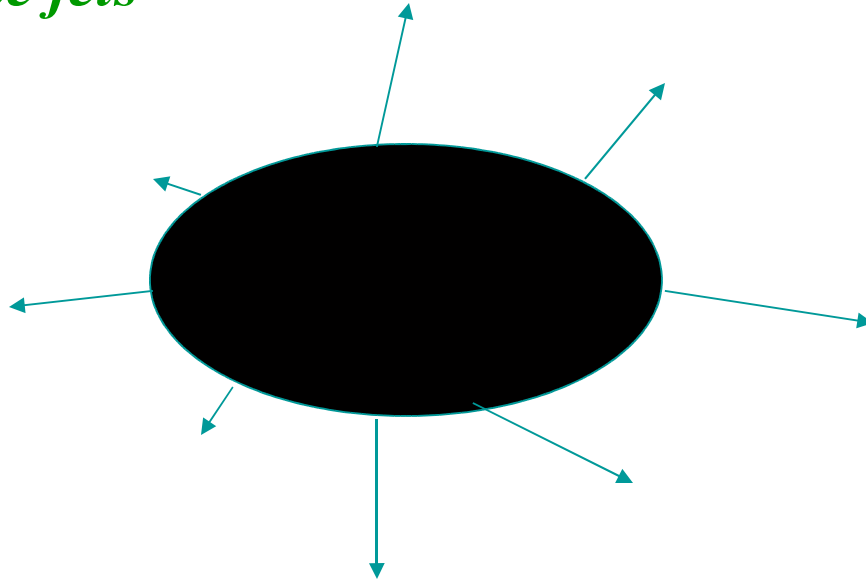


$$z_1 = \frac{p_1^+}{p^+}$$

$$z_2 = \frac{p_2^+}{p^+}$$

## ***SURFACE EMISSION PICTURE***

- *Suppose the matter produced is very opaque*
- *Hence only hard collisions on the surface will produce observable jets*



- *Inconsistent with an  $R_{AA}$  near participant scaling*
- *Inconsistent with all energy loss models which require bulk emission and fit single inclusive data!*

# *How does partonic interaction effect dihadrons?*

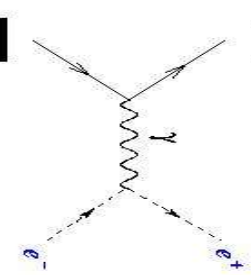
- *Important to  $A+A$ ,  $d+A$ , DIS and  $e^+ e^-$  experiments.*
- *To date observations in  $A+A$   $d+Au$  and DIS*

## *Wish List!*

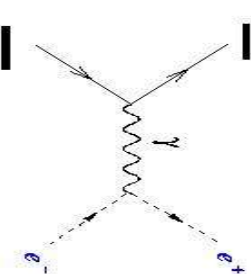
- *Definition and factorization of fragmentation functions*
- *Calculate the effect of medium modification*
- *Requires the evaluation of twist 4 diagrams,*
- *But medium modification similar to vacuum evolution*
- *Calculate and check vacuum evolution first (simpler!)...*

# Basic Methodology at L. O. in $\alpha_s$ :

*Replace partonic basis with hadronic basis*

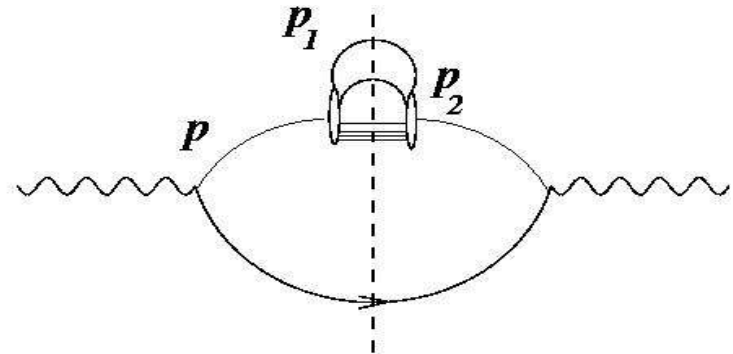
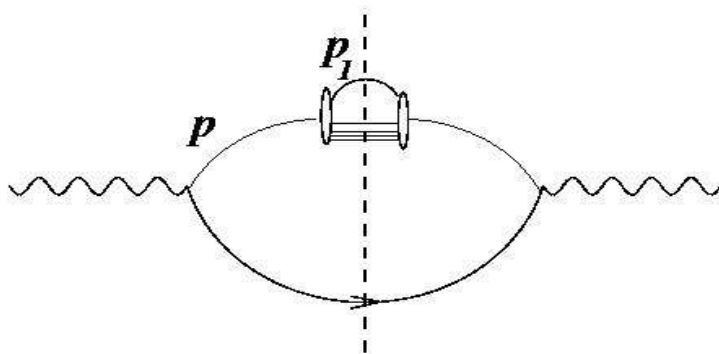
$$D_q^h(z) \longrightarrow \sum_{S-1} \left| \begin{array}{c} \langle k | \\ |R_h, S-1\rangle \end{array} \right| \quad \mathbf{2}$$


A Feynman diagram showing a quark line (dashed) with momentum  $q$  entering from the bottom left and exiting from the bottom right. A gluon line (wavy) with momentum  $k$  is emitted from the quark line. The gluon line then splits into two quark lines (dashed) with momenta  $q$  and  $q$  entering from the bottom left and bottom right respectively. The top vertex is labeled  $|R_h, S-1\rangle$ .

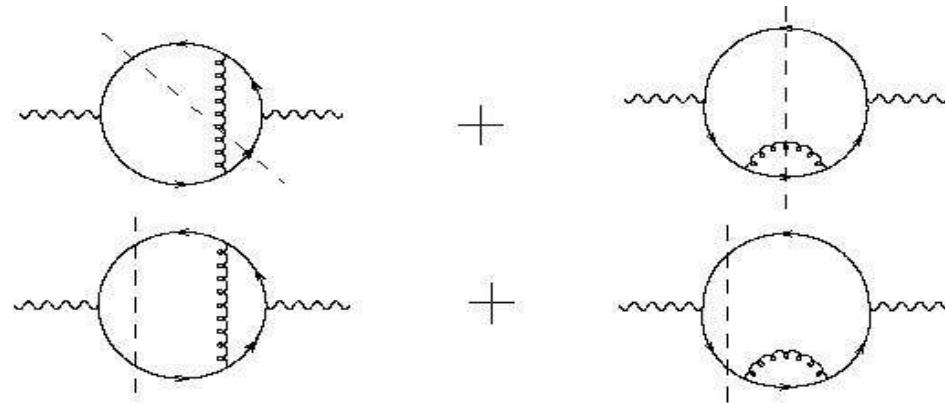
$$D_q^{h_1 h_2}(z_1, z_2) \longrightarrow \sum_{S-2} \left| \begin{array}{c} \langle k | \\ |R_1 R_2, S-2\rangle \end{array} \right| \quad \mathbf{2}$$


A Feynman diagram showing a quark line (dashed) with momentum  $q$  entering from the bottom left and exiting from the bottom right. A gluon line (wavy) with momentum  $k$  is emitted from the quark line. The gluon line then splits into two quark lines (dashed) with momenta  $q$  and  $q$  entering from the bottom left and bottom right respectively. The top vertex is labeled  $|R_1 R_2, S-2\rangle$ .

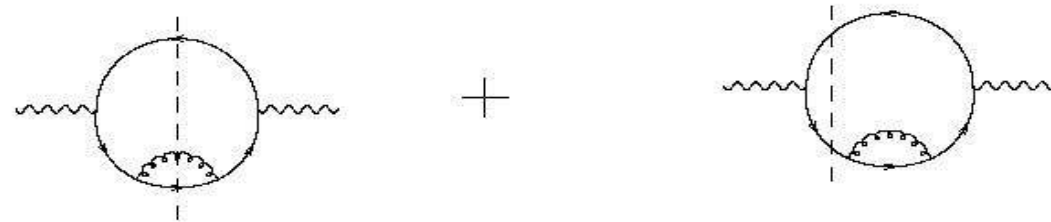
*In self-energy diagrams*



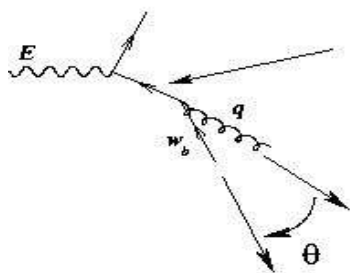
- *Isolation of matrix element is the first step of factorization*
- *Calculate higher order corrections and isolate leading log and power contributions. Isolate the hard part.*
- *Leading log contributions have divergences, absorb into fragmentation functions*



*In Light cone gauge, leading log contributions only come from*

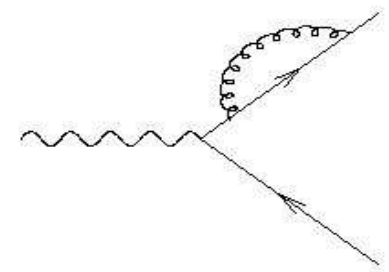


*Infrared and collinear divergences from soft and collinear gluons*



$$\frac{1}{(1 - \cos\theta) q w_b}$$

*Infrared divergences cancelled by self energy diagrams*



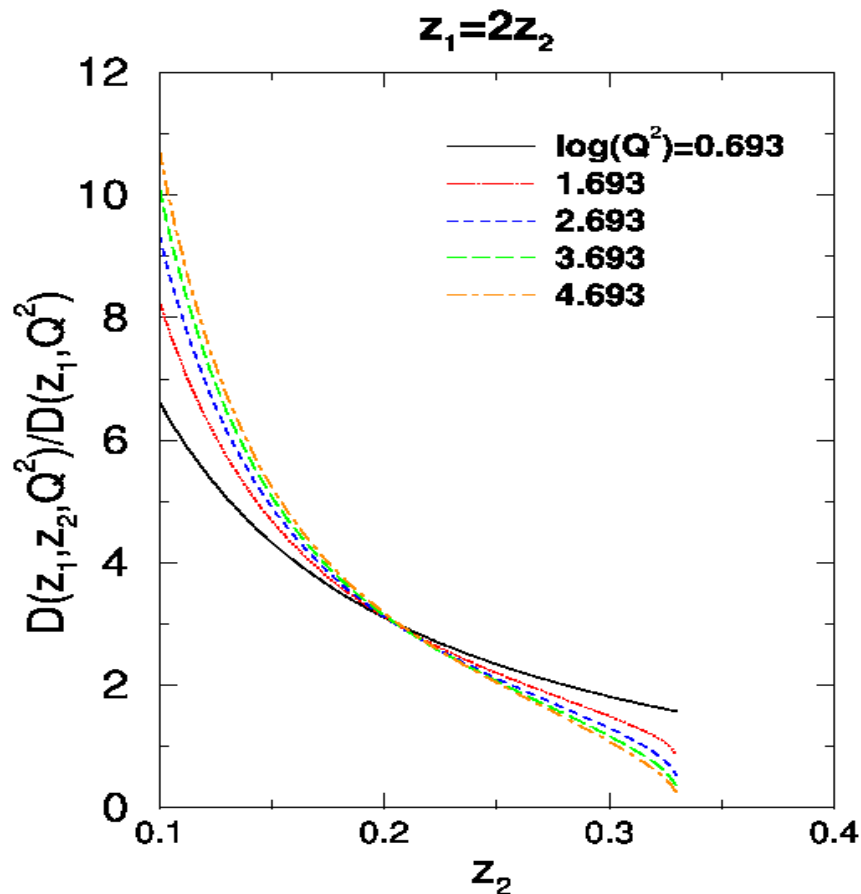
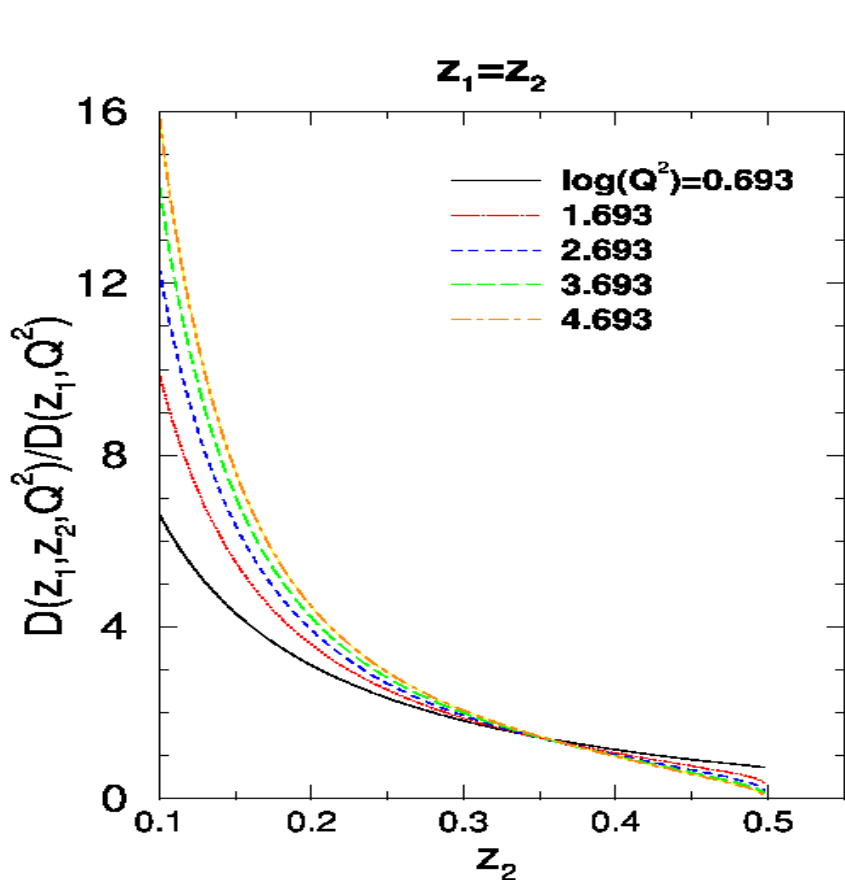
# Use factorized distribution for Non-singlet fragmentation function

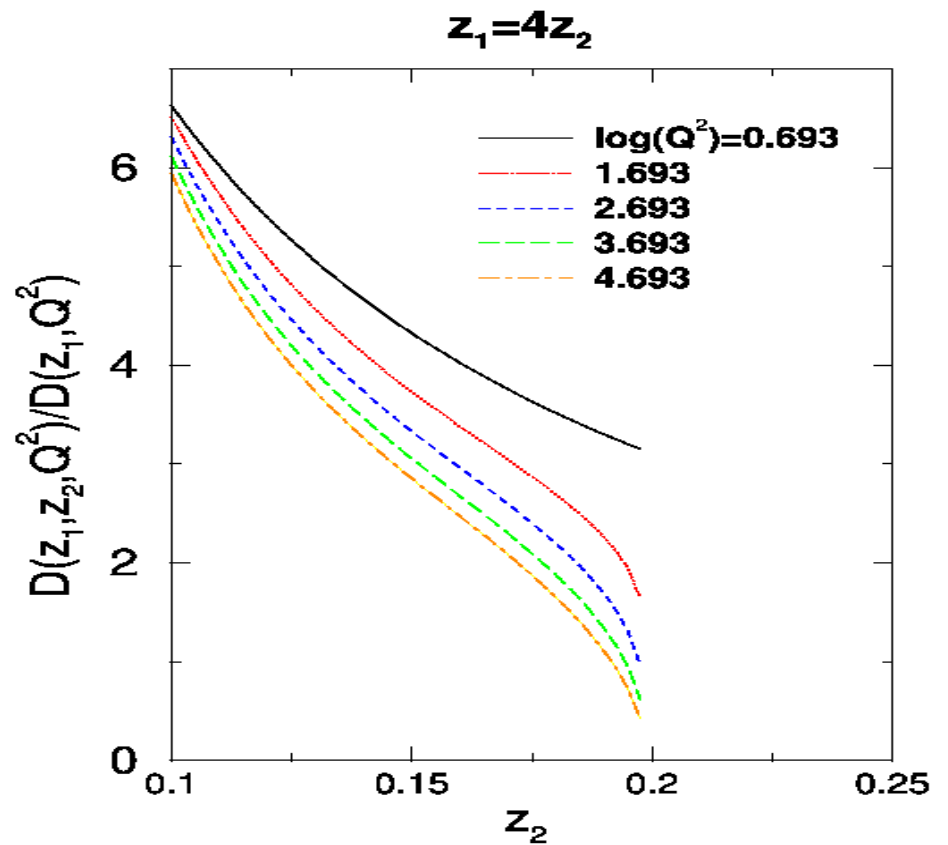
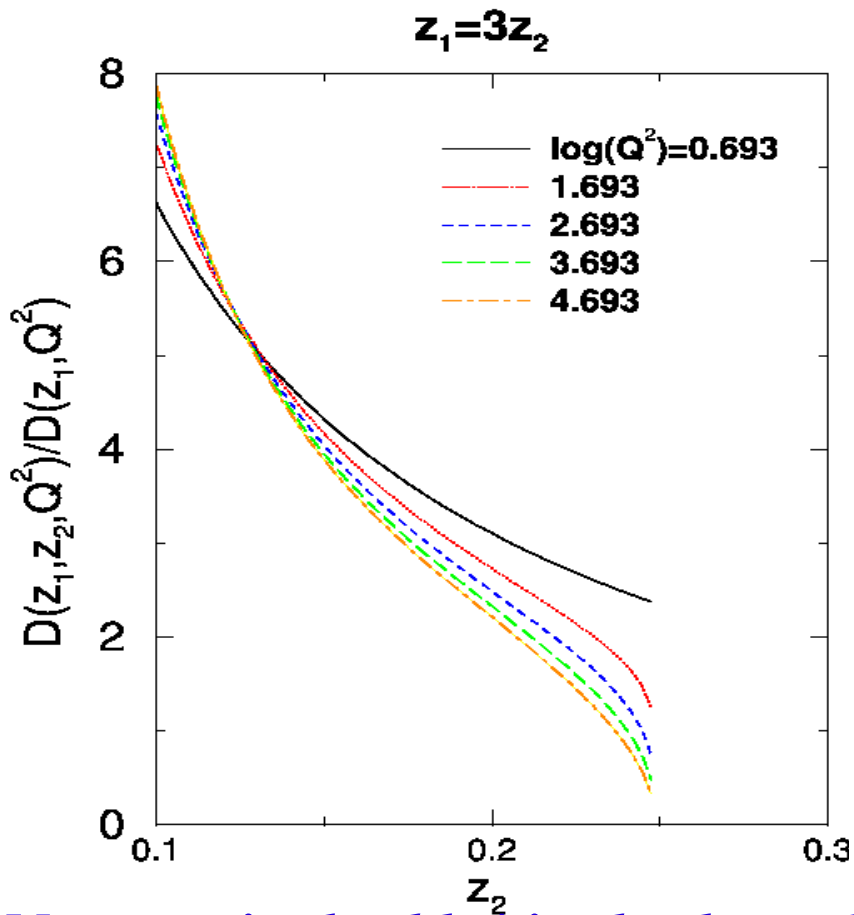
$$\text{Factorized Distribution: } D(z_1, z_2, \mu) = D(z_1, \mu) D(z_2, \mu)$$

Single fragmentation functions taken from KKB

Plots for  $z_1/z_2 = 1, 2, 3, 4$

$$\log(Q^2 = 2 \dots 110 \text{ GeV}^2) = 0.693 \dots 4.693$$





*Note: ratio double/single shows little change at intermediate  $z$ . Why ?*

*Ratio is the number of associated particles for given trigger !*

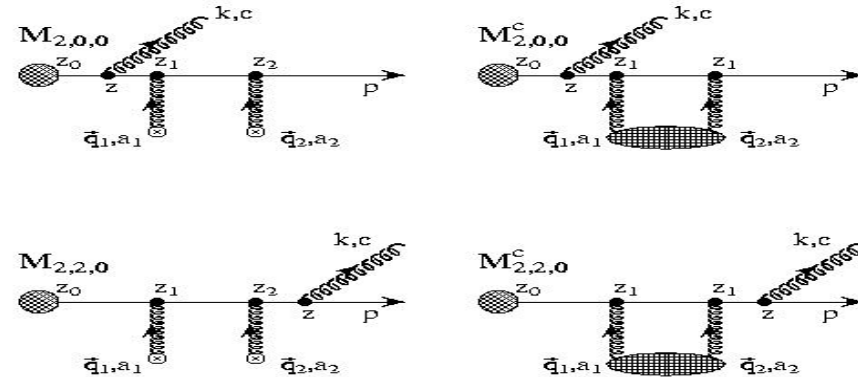
*Regular evolution softens the spectrum: as for single hadrons*

*Single gluon fragmentation increases multiplicity!*

# CALCULATION OF THE MODIFICATION

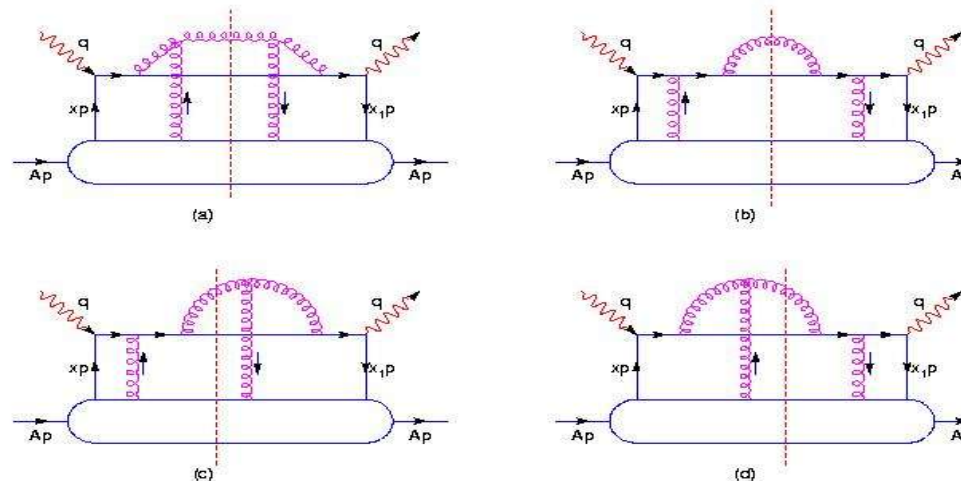
1) *Potential scattering model : Gyulassy Wang; Gyulassy Levai Vitev; Weidemann Salgado..*

*Scattering of static color sources, e-loss by gluon radiation, followed by radiation reinteraction..*



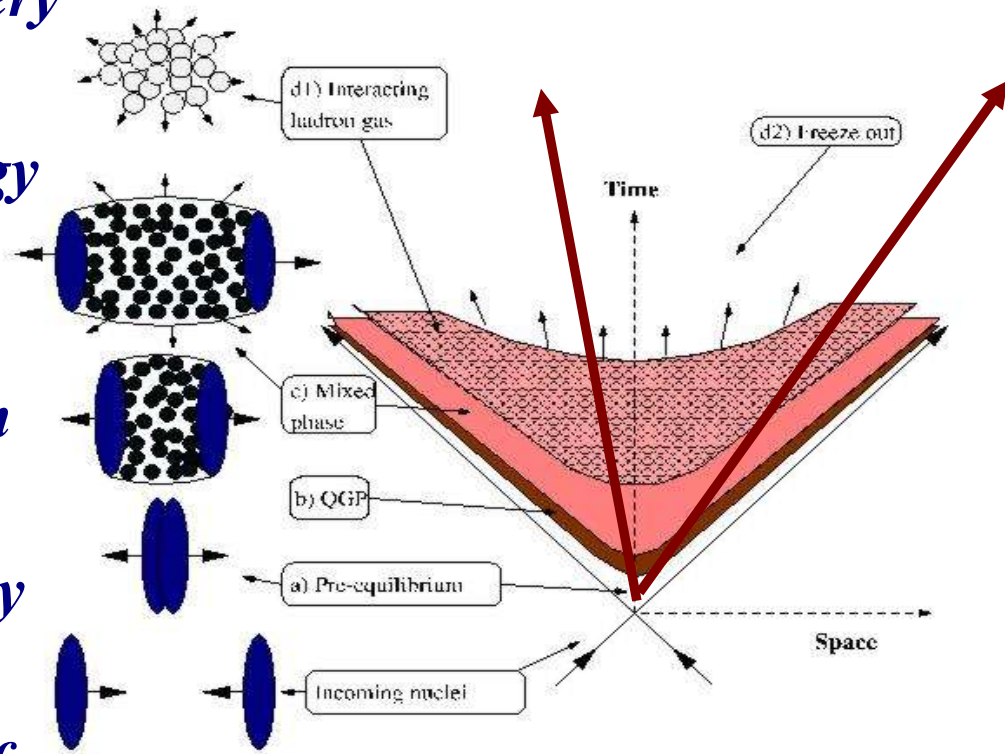
2) *Modification of fragmentation function by higher power corrections from the medium: Guo Wang; Osborne Wang..*

*Based directly on DIS formalism,  
Power corrections from structure functions enhanced by size!*



# HEAVY-ION COLLISIONS AND JETS

- Collide 2 heavy ions at very high energy.
- Given high enough energy density a QGP may be created.
- But QGP turns to hadron gas and freezes out.
- Occasionally, high energy jets produced.
- Jets sample the history of the collision.
- Study of jet properties may produce insight into matter produced



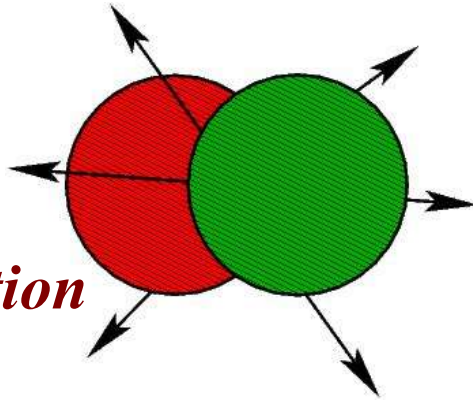
*Compare with jet properties in p-p or electron positron annihilation*

*And with modification in cold nuclear matter*

# SINGLE PARTICLE MEASUREMENTS in heavy-ion coll.

High  $P_T$  particle production at midrapidity

Nuclear modification factor  $R_{AA}$

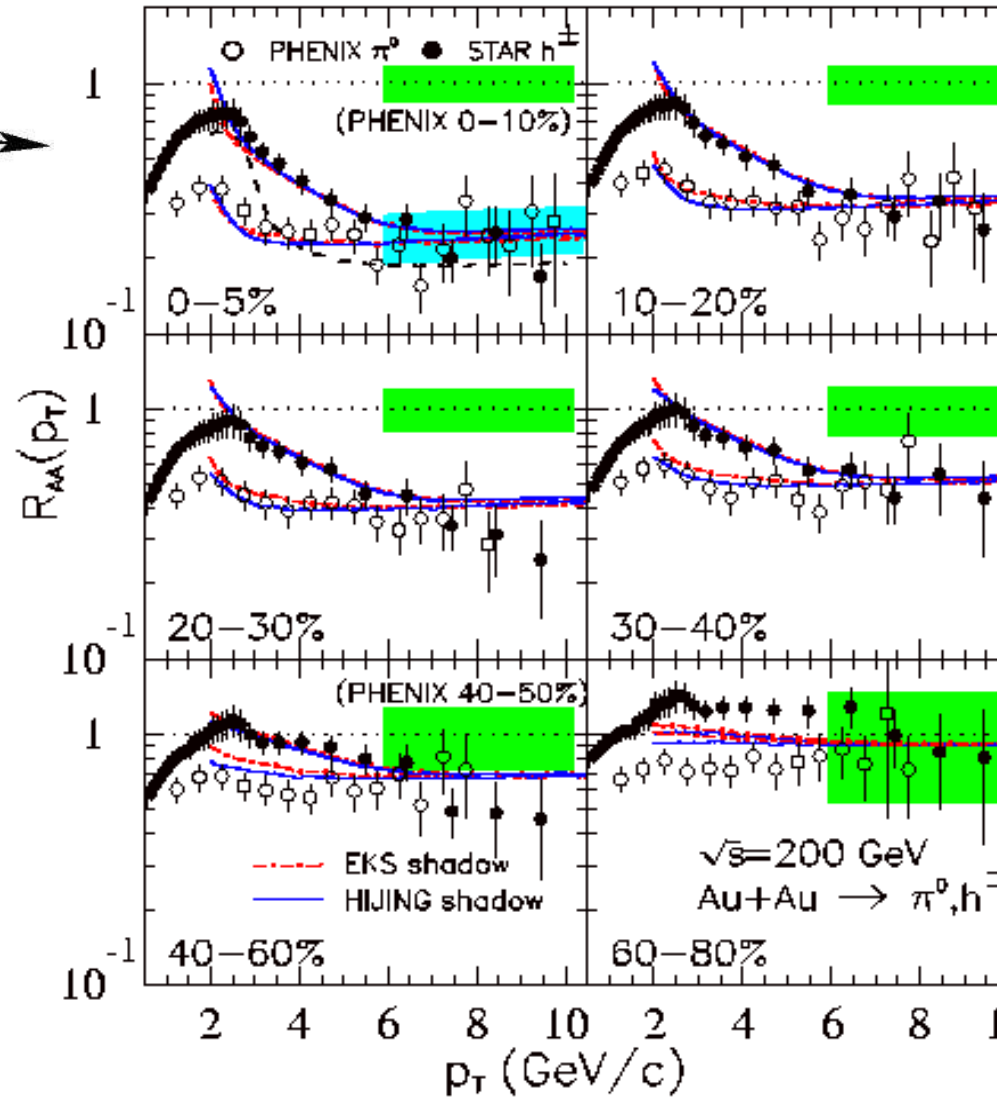


$$R_{AA} = \frac{d^2 N_{AA} / d\eta dp_{\perp}}{(N_{coll} / \sigma_{pp}^{inelastic}) d^2 \sigma / d\eta dp_{\perp}}$$

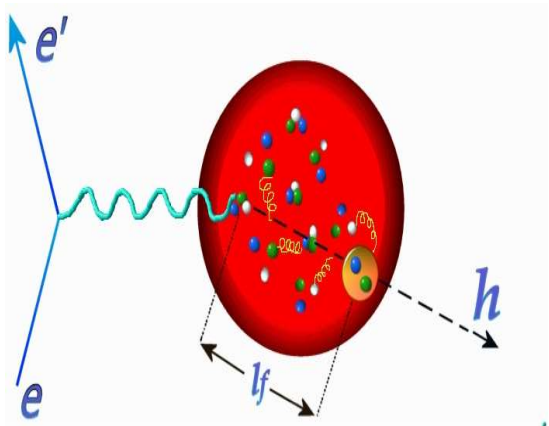
$1$  = particle production scales with number of expected  $p$ - $p$  collisions

includes initial state modification of structure functions

11/12/04



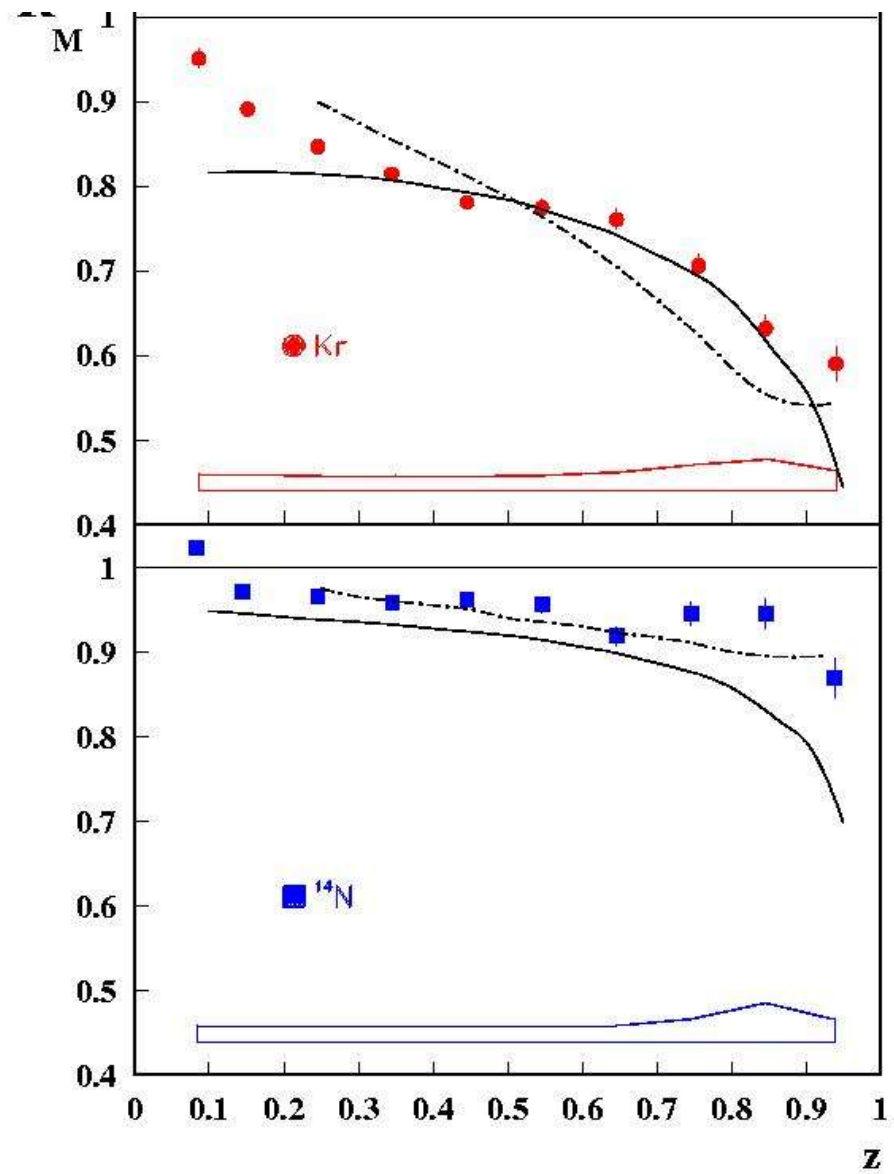
# Single hadron attenuation in Deep-Inelastic Scattering



*Perform DIS of nuclei*

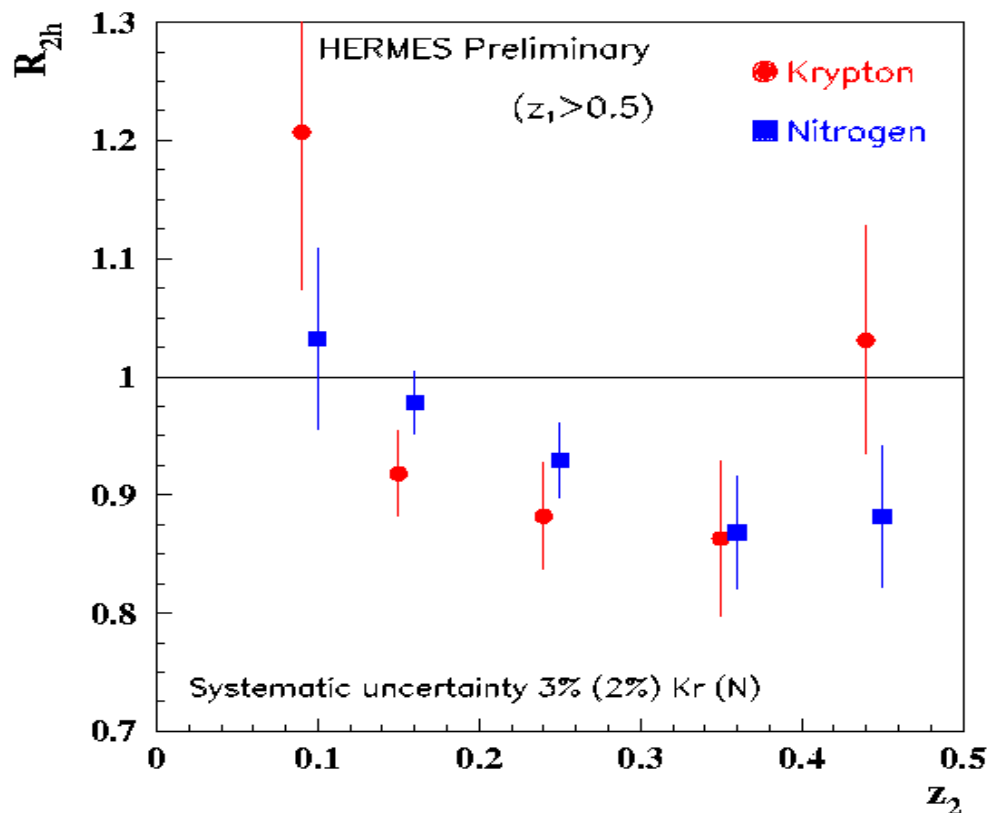
*Look at particle production in the forward region*

*compare with DIS off light nuclei: Deuterium*



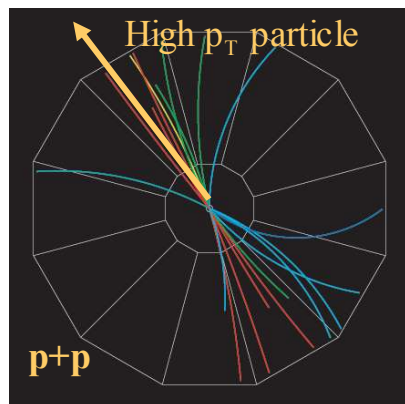
# Double hadron measurements in DIS

- *Always measure a ratio of double to single production*
- *Divide by same ratio in deuterium to remove detector systematics*
- *As in Heavy-ion collisions very little change of double/single ratio.*

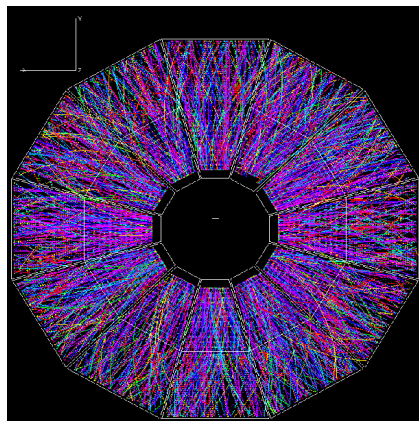


$$R_{2h} = \frac{\text{No. of events with at least 2 hadrons with } z_1 > 0.5}{\frac{\text{No. of events with at least one hadron with } z > 0.5}{\text{same ratio on deuterium}}}$$

# *SINGLE HADRONS AND DI-HADRONS*



- *1 particle inclusive production, factorize from hard cross section*  $\Rightarrow D(z)$  *fragmentation function*
- *Measure 2 particle distribution*  $\Rightarrow D(z_1, z_2)$

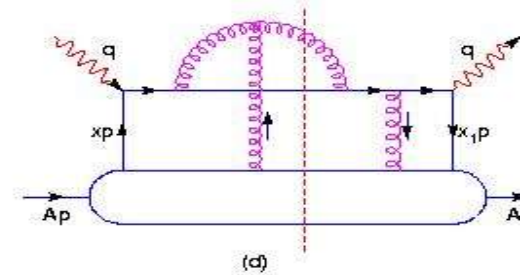
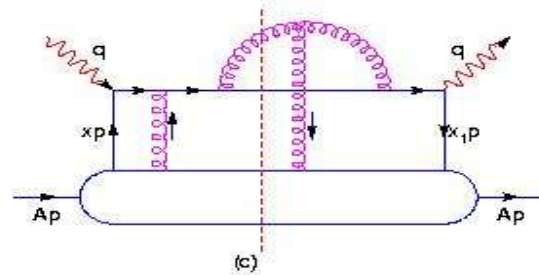
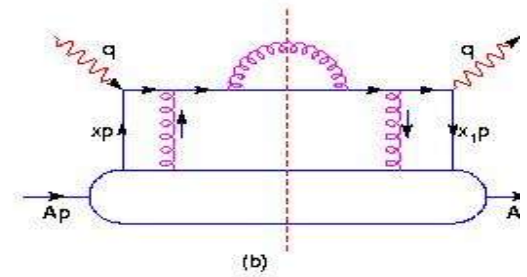
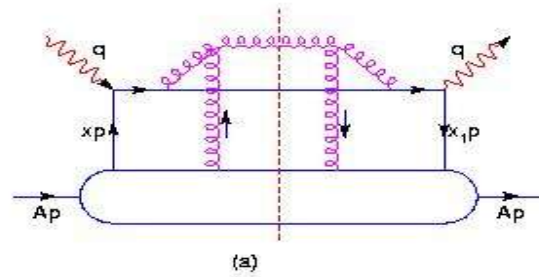


- *Can do single inclusive measurements*
- *Can still do 2-particle measurements*
- *Select a leading particle  $4 < p_t < 6 \text{ GeV}/c$ ,  $|\eta| < 0.75$*
- *Associate all other particles ( $0.15 < p_t < 4 \text{ GeV}/c$ ,  $|\eta| < 1.1$ ) with the leading particle.*

# Multiple higher twist diagrams need to be evaluated

Multiple scattering  
from soft gluons lead to  
LPM interference

Assume a Gaussian  
density distribution  
for nucleons in a  
medium sized nucleus



$$T_{qg}^A = C A^{1/3} (x G^N(x)) (1 - e^{-x_L^2/x_A^2})$$


$$\frac{x_L^2}{x_A^2} = \frac{R_A^2}{\gamma^2 \tau_f^2}$$

$\tau_f$  = Formation time

$R_A$  = Nuclear size

$\gamma$  = boost

# OUTLINE

- **MOTIVATIONS: HEAVY-ION COLLISIONS AND QGP, DIS**
- **VARIOUS APPROACHES: MEDIUM MODIFICATION OF FRAGMENTATION FUNCTIONS VIA PARTONIC INTR.**
- **DEFINITION OF DIHADRON FRAGMENTATION**   $e^+ e^-$
- **DGLAP EVOLUTION: MODIFICATION IN VACUUM**
- **MEDIUM MODIFICATION IN COLD MEDIUM (DIS)**
- **MEDIUM MODIFICATION IN HOT MEDIUM (QGP)**

## *Simpler case of Non-singlet quarks*

$$D_{NS} = D_q - D_{\bar{q}}$$

*Simpler: contribution from gluon fragmentation cancels out*

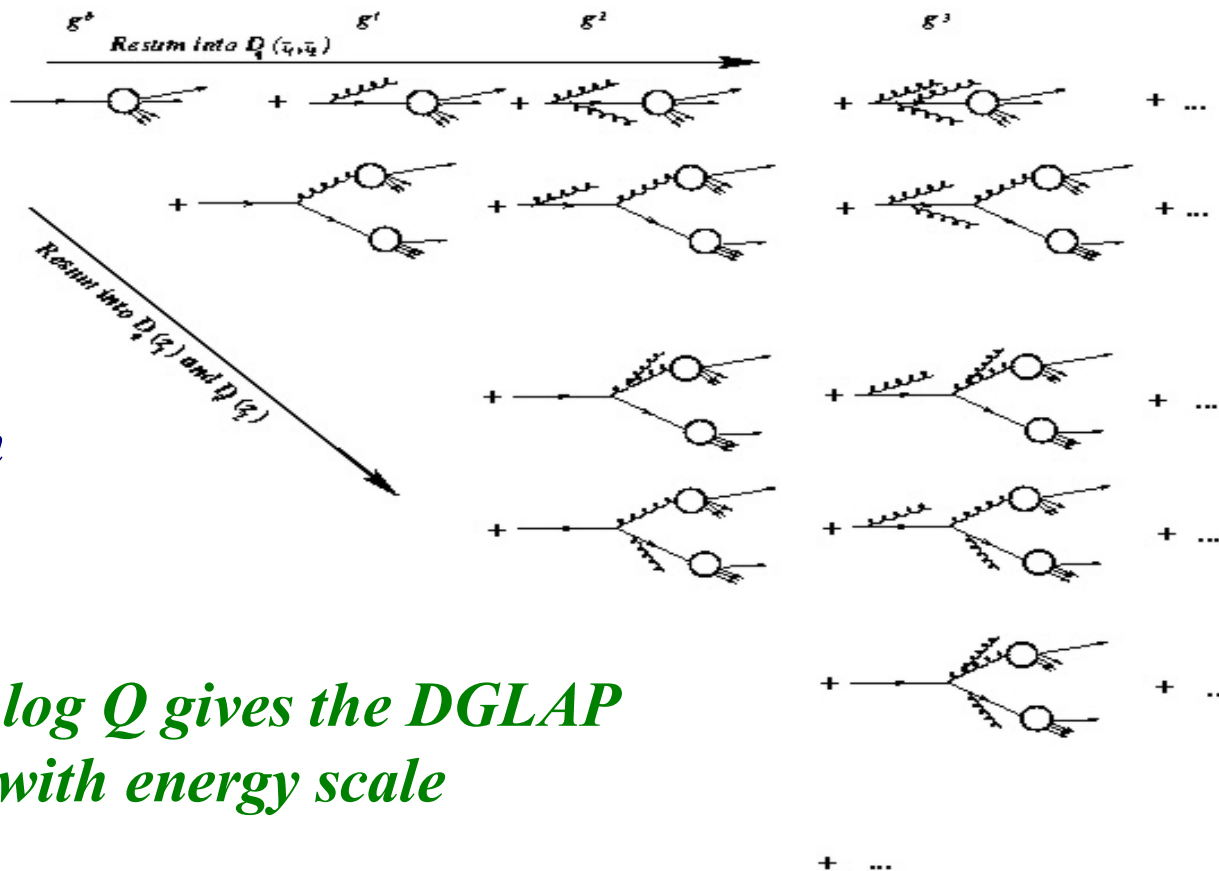
$$D_{NS}(z_1, z_2, \mu^2) = D_{NS}(z_1, z_2) + \int_0^{\mu^2} \frac{dl_{\perp}^2}{l_{\perp}^2} \int_z^1 \frac{dy}{y^2} \left[ \frac{1+y^2}{1-y} \right]_+ D_{NS}(z_1/y, z_2/y) \\ + \int_0^{\mu^2} \frac{dl_{\perp}^2}{l_{\perp}^2} \int_{z_1}^{1-z_2} \frac{dy}{y(1-y)} \frac{1+y^2}{1-y} D_q(z_1/y) D_g(z_2/(1-y))$$

- ◆ *Absorb collinear divergences up to a scale  $\mu$*
- ◆ *Defines fragmentation function at scale  $\mu$*
- ◆ *Note: only retained leading log and leading power corrections*
- ◆ *To go to a higher scale this has to be repeated to all orders*
- ◆ *Leading log corrections absorbed at each order*

*To obtain the dihadron fragmentation function at a higher scale  $Q$  need to resum the LL contributions from all orders as shown...*

*LL contributions  
can be resummed*

*part of the LL  
contributions  
resummed into the  
single fragmentation  
functions...*



*Differentiating with  $\log Q$  gives the DGLAP  
evolution equations with energy scale*

*Each differentiation extracts a splitting  
function, and reduces the number of gluons*