Deep inelastic lepton nucleus scattering and hadronization

H.J. Pirner, D. Grünewald, A. Accardi und V. Muccifora,

Universität Heidelberg,Iowa State Univ., INFN - Frascati

A.Accardi, V. Muccifora and H.J. Pirner Nucl.Phys. A720 (2003) 131, and Paper by the same authors with D. Grünewald in preparation

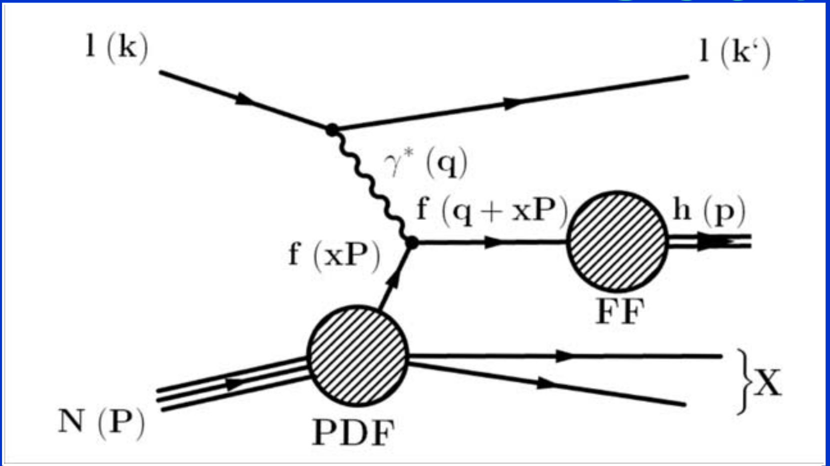
Outline

- Motivation for semi inclusive deep inelastic lepton nucleus scattering
- Building blocks of the absorption model
- Rescaling of PDF and FF
- LUND string fragmentation model -> formation length
- Absorption factor
- Comparison with HERMES data
- Mass number dependence of attenuation
- Conclusions

Motivation

- Photon Quark Scattering gives a well defined starting point of hadronization
- Medium in which the quark propagates is well known: cold nuclear matter
- Soft processes dominate at Hermes $\langle Q^2 \rangle = 2.5\ GeV^2$
- We can compare energy loss of the quark with absorption of a colorless prehadron

Semi Inclusive deep inelastic scattering



Variable	Covariant	Lab. frame
Q^2	$-q^2$	$2Mx\nu$
ν	$\frac{qp}{\sqrt{P^2}}$	$E'-E$
x	$\frac{-q^2}{2Pq}$	$\frac{Q^2}{2M\nu}$
z	$\frac{pP}{qP}$	$\frac{E_h}{\nu}$
y	$\frac{qP}{kP}$	$\frac{\nu}{E}$
W^2	$(P+q)^2$	$M^2 + 2M\nu - q^2$

- Factorization theorem in QCD:

$$\left.\frac{d^2\sigma}{dx\,d\nu\,dz}\right|_{SIDIS} = \sum_f e_f^2 q_f(x,Q^2)\frac{d^2\sigma^{lq}}{dx\,d\nu}D_f^h(z,Q^2)$$

- Multiplicity:

$$M^h(z) = \frac{1}{N_A^{DIS}}\frac{dN_A^h(z)}{dz}$$

$$\frac{1}{N^{DIS}}\frac{dN^h(z)}{dz} = \frac{1}{\sigma^{lp}}\int_{\text{exp. cuts}} dx\,d\nu\sum_f e_f^2 q_f(x,Q^2)\frac{d\sigma^{lq}}{dx\,d\nu} \times D_f^h(z,Q^2)$$

$$\sigma^{lp} = \int_{\text{exp. cuts}} dx\,d\nu\sum_f e_f^2 q_f(x,\xi_A(Q^2)Q^2)\frac{d\sigma^{lq}}{dx\,d\nu}$$

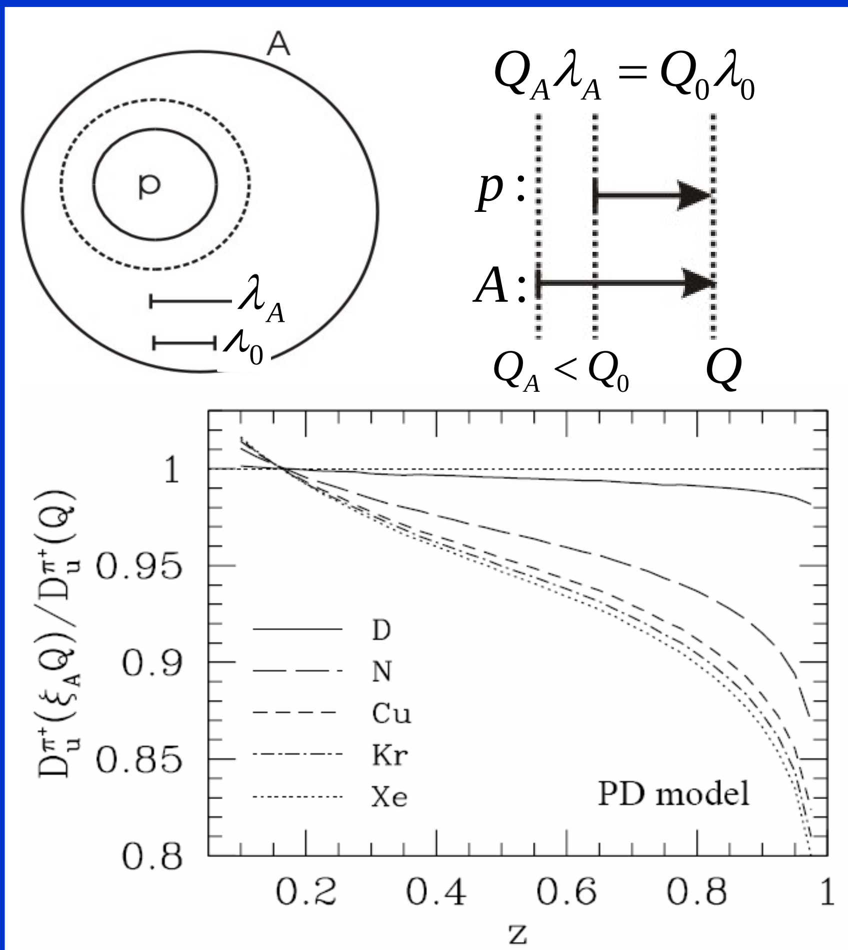
Building Blocks of the Absorption Calculation

$$\frac{1}{N_A^{DIS}}\frac{dN_A^h(z)}{dz} = \frac{1}{\sigma^{lA}} \int_{\text{exp. cuts}} dx d\nu \sum_f e_f^2 q_f^A(x, \xi_A Q^2) \frac{d\sigma^{lq}}{dx d\nu}$$

$$\times D_f^h(z, \xi_A Q^2) N_A(z, \nu),$$

Rescaling of Parton Distribution, Rescaling of Fragmentation Function
Calculation of the mean formation times of the prehadron and hadron
Calculation of the Nuclear Absorption Factor N_A,using formation times

Rescaling of PDF and FF



- Assume change of confinement scale in bound nucleons $\lambda_A > \lambda_0$
- Two consequences:

1.)

$$\frac{1}{A} q_f^{N|A}(x, Q^2) = q_f^N(x, \xi_A(Q^2) Q^2)$$

$$D_f^{h|A}(z, Q^2) = D_f^h(z, \xi_A(Q^2) Q^2)$$

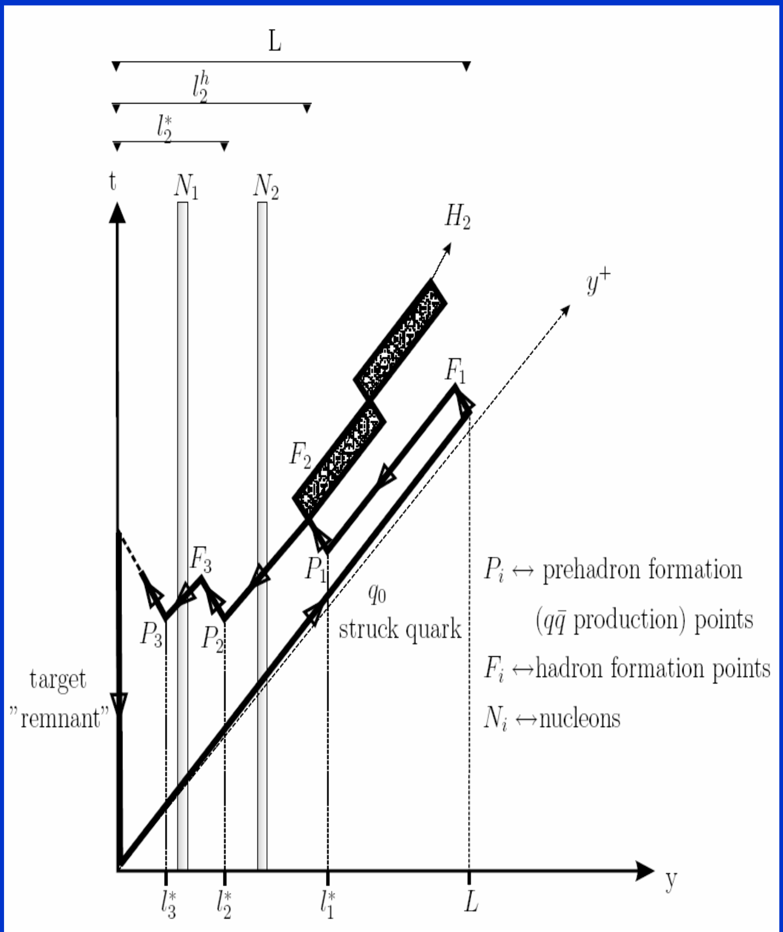
$$\xi_A(Q^2) = \left(\frac{\lambda_A}{\lambda_0}\right)^{\frac{\bar{\alpha}_s}{\alpha_s(Q^2)}}$$

2.)

$$\kappa_A \lambda_A^2 = \kappa \lambda_0^2$$

- Rescaling implies a longer DGLAP evolution (increased gluon shower)

Standard LUND String Fragmentation Model



- **First rank particle contains struck quark -> flavor dependent formation length**
- **String fragmentation function:**

  $f(u) \propto (1-u)^{D_a}$ $\quad D_q = 0.3$ and $D_{qq} = 1.3$

  proportional to $\exp\left(-\frac{\pi\mu^2}{\kappa}\right)$

  -> dominantly quark production

- **Turning point of struck quark:**

  $L = \frac{\nu}{\kappa}$ $\quad \kappa = 1GeV/fm$

- **Consider renormalization of string tension due to realistic confinement scales of hadrons:** $L_h = \frac{\nu}{\kappa}\frac{r_h^2}{r_\pi^2}$

1st and 2nd Generation K-Production

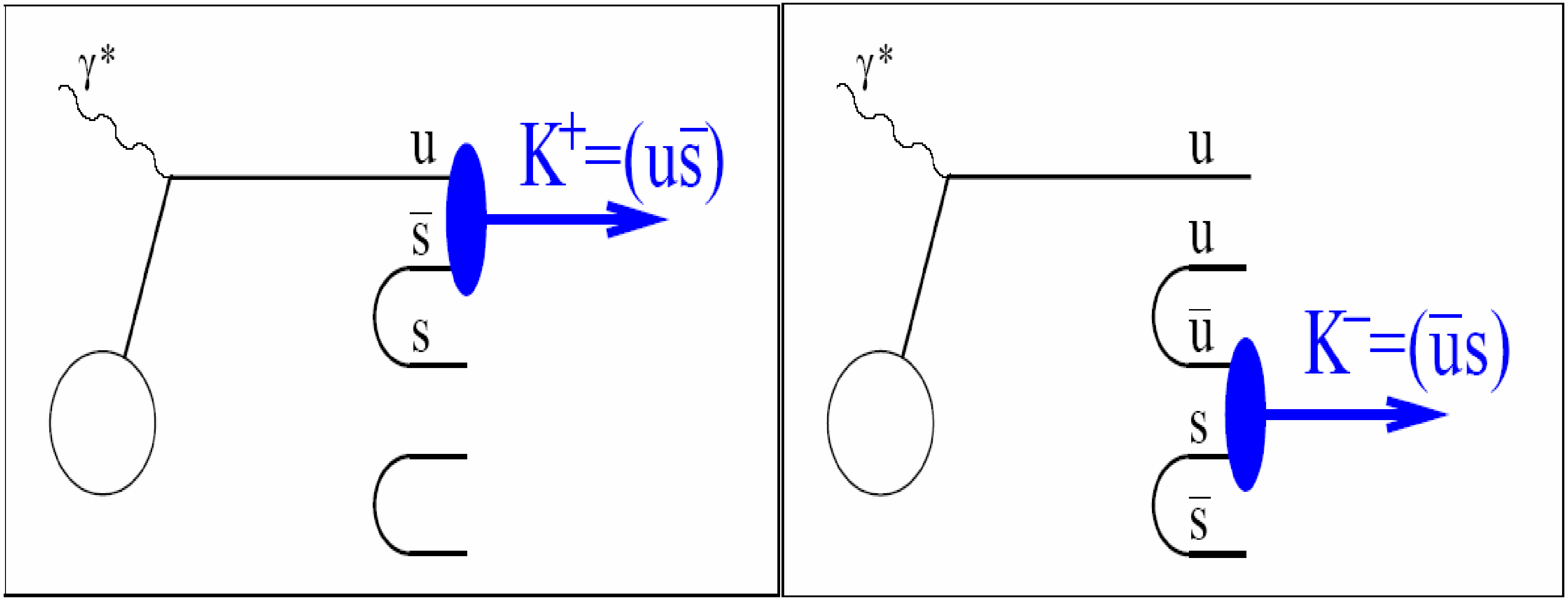


Fig. 2. Schematic picture describing the fragmentation of an up quark into K^+ and K^-. A similar picture works also for fragmentation into p and $\bar{p}$, respectively.

Calculation of Prehadron Formation Lengths (1)

$$\langle l^*_{\geq 1} \rangle = \frac{1+D_a}{1+C+(D_a-C)z}(1-z)\,z\,L$$

$$\times\left[1+\frac{1+C}{2+D_a}\frac{(1-z)}{z^{2+D_a}}\,{}_2F_1\left(2+D_a,2+D_a;3+D_a;\frac{z-1}{z}\right)\right]$$

F- Hypergeometric Function, C=0.3, D arise from the string fragmentation f(u)=(1-u)^D

Prehadron Formation Lengths

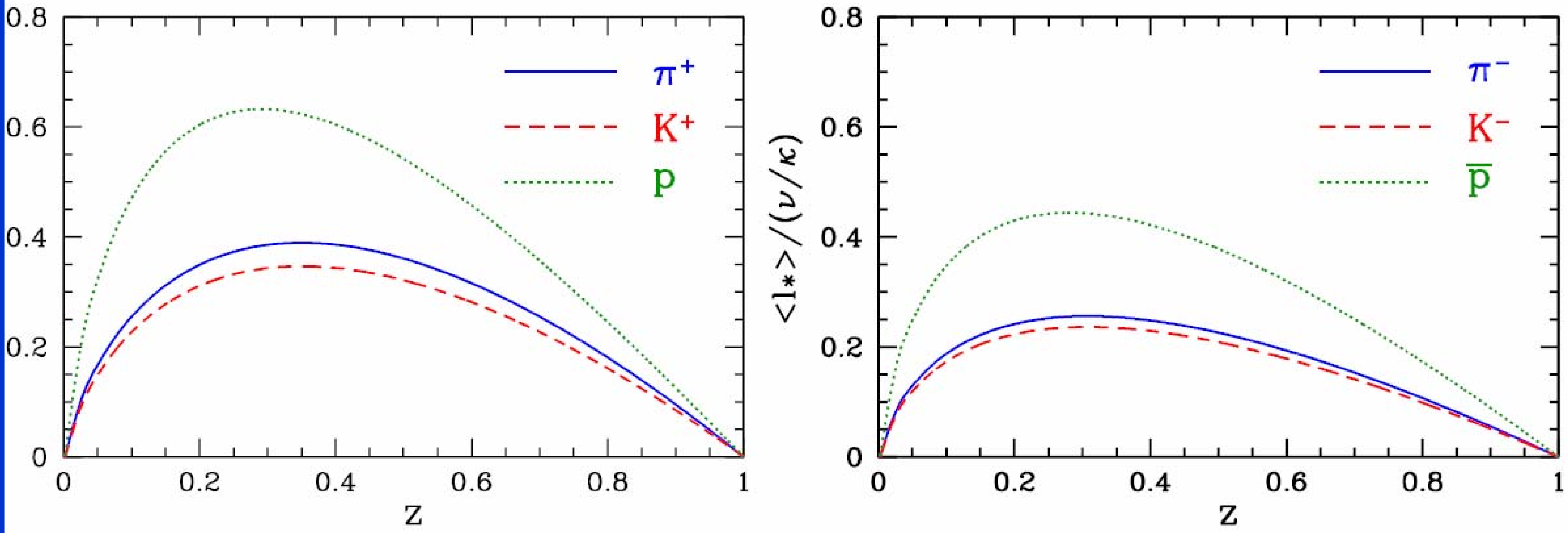


Fig. 3. Computed prehadron formation lengths when an up quark is struck by the virtual photon. *Left:* When a π^+, K^+ or p is observed, the corresponding prehadron can be created at rank $n \geq 1$. *Right:* When a π^-, K^- or $\bar{p}$ is observed, the corresponding prehadron can be created only at rank $n \geq 2$.

Scaled Hadron f.l.=p.f.l.+z

Absorption model

- Inelastic scattering of (pre)hadrons on nucleons removes them from the considered (z,nu) bin, rate is determined by mean free path

$$\frac{\partial P_q(y,y')}{\partial y'} = -\frac{P_q(y,y')}{\langle l^* \rangle} \qquad , P_q(y,y'=y)=1$$

$$\frac{\partial P_*(y,y')}{\partial y'} = \frac{P_q(y,y')}{\langle l^* \rangle} - \frac{P_*(y,y')}{\langle \Delta l \rangle} - \frac{P_*(y,y')}{\lambda_*(y')} \qquad , P_*(y,y'=y)=0$$

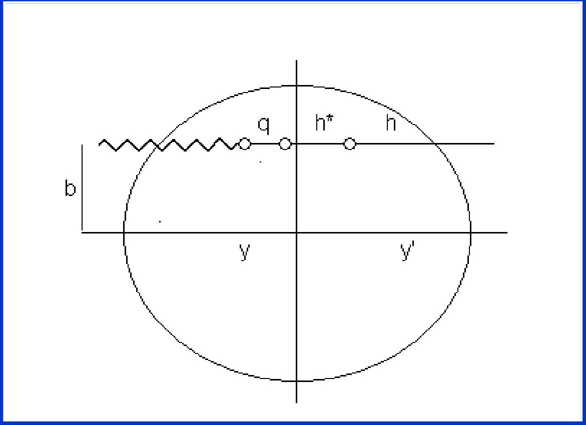
$$\frac{\partial P_h(y,y')}{\partial y'} = \frac{P_*(y,y')}{\langle \Delta l \rangle} - \frac{P_h(y,y')}{\lambda_h(y')} \qquad , P_h(y,y'=y)=0$$

- Absorption factor:

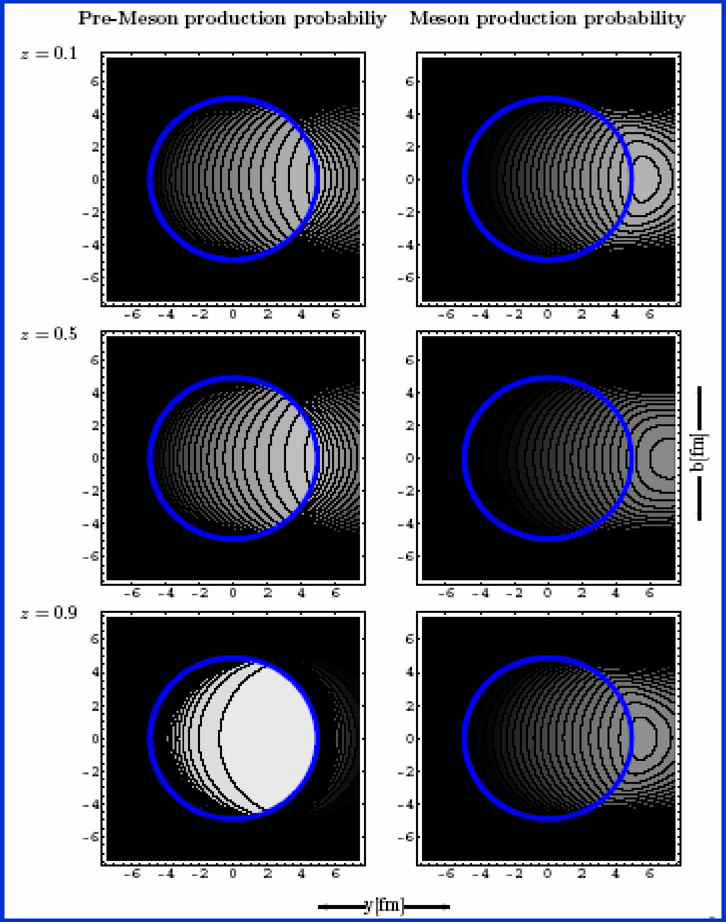
$$N_A = \lim_{y' \to \infty} \int d^2b \int_{-\infty}^{\infty} dy \rho_A(b,y) P_h(y',y)$$

$$= \int d^2b \int_{-\infty}^{\infty} dy \rho_A(b,y) \int_y^{\infty} dx' \int_y^{x'} dx \frac{e^{-\frac{x-y}{\langle l^* \rangle}}}{\langle l^* \rangle} e^{-\sigma_* \int_x^{x'} ds A \rho_A(s)}$$

$$\times \frac{e^{-\frac{x'-x}{\langle \Delta l \rangle}}}{\langle \Delta l \rangle} e^{-\sigma_h \int_{x'}^{\infty} ds A \rho_A(s)}$$

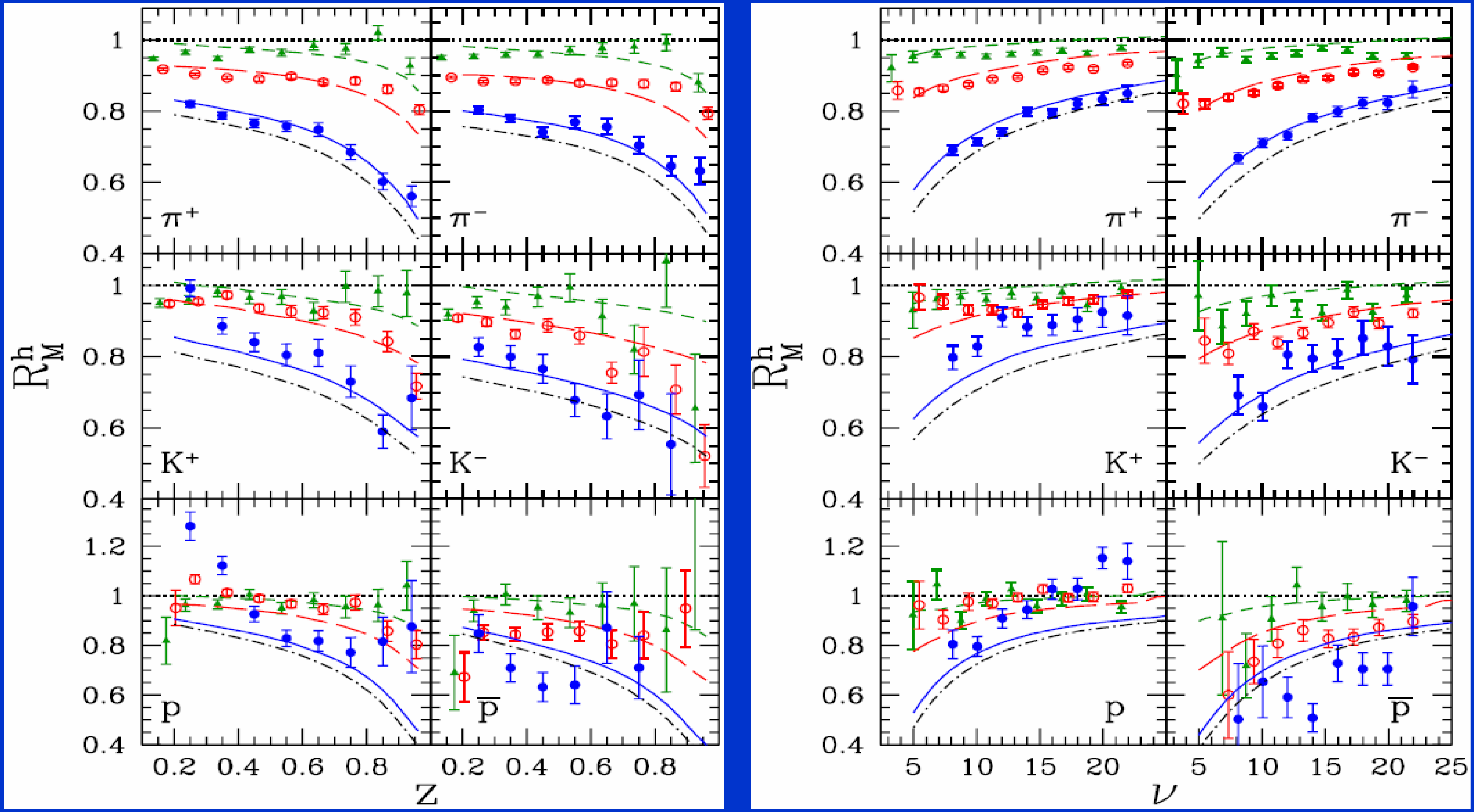


Prehadron und Hadron-Production probabilities at HERMES energies for Kr target without absorption



Comparison with HERMES data

Hermes Coll. A.Airapetian et al. Phys. Lett. B577 (2003) 37-Xe,Kr,Ne,He target



Intermediate Resumé:

- Rescaling + absorption are able to describe the data with $\sigma_*(\mathrm{prehadron})=2/3\sigma(\mathrm{hadron})$
- Flavor dependent formation length is necessary
- Proton description is still unsatisfactory

  because rescatterings may shift z to lower values and/or higher twist diquark scattering

Naive A-dependence of attenuation

- Absorption model:

  due to the fact that the attenuation is proportional to the in-medium path length traversed by the hadron

  1-R_M=c A<u>^(1/3)</u>

  Energy loss model:

  due to the fact that the average energy loss is proportional to the in-medium path length <u>squared</u>

  1-R_M=c <u>A^(2/3)</u>

- But!!! ….

A dependence of attenuation Absorption model

- Consider only prehadron formation
- Hard sphere nucleus
- Analytical calculation becomes possible
- W=0.19

independently of $a = \frac{\langle l^* \rangle}{\lambda}$. Integrating the approximate integrand, one finds a rapidly converging expansion of the attenuation, since now the small value of the w-parameter enhances the convergence:

$$\begin{aligned} 1 - R_M &= \frac{1}{5} wab^2 - \frac{3}{70}(wab^2)^2 + \mathcal{O}[(wab^2)^3] \\ &= c_1 A^{2/3} - c_2 A^{4/3} + \mathcal{O}[A^2] \ . \end{aligned} \tag{26}$$

A dependence of attenuation
Absorption model

(hard sphere nucleus $R = r_0 A^{1/3}$)

- 1-step absorption:

$$\frac{\partial P_q(y,y')}{\partial y'} = -\frac{P_q(y,y')}{\langle l^* \rangle} \quad , P_q(y,y'=y)=1$$

$$\frac{\partial P_*(y,y')}{\partial y'} = \frac{P_q(y,y')}{\langle l^* \rangle} - \frac{P_*(y,y')}{\langle \Delta l \rangle} - \frac{P_*(y,y')}{\lambda_*(y')} \quad , P_*(y,y'=y)=0$$

$$\frac{\partial P_h(y,y')}{\partial y'} = \frac{P_*(y,y')}{\langle \Delta l \rangle} - \frac{P_h(y,y')}{\lambda_h(y')} \quad , P_h(y,y'=y)=0$$

- Neglect Deuterium:

$$1 - R_M = 1 - \frac{\pi \rho_0}{A} \int_0^{R^2} db^2 \int_{-R(b)}^{R(b)} dy \int_y^{\infty} dx \frac{e^{-\frac{x-y}{\langle l^* \rangle}}}{\langle l^* \rangle} e^{-\rho_0 \sigma_* \int_x^{\infty} ds \Theta(R(b)-|s|)}$$

- Some simplifications:

$$1 - R_M = \frac{\pi \rho_0}{2A} \langle l^* \rangle^3 \int_0^{2R/\langle l^* \rangle} dt t \int_0^t dr \int_0^r du e^{-u} \left[1 - e^{\frac{\langle l^* \rangle}{\lambda_0}(u-r)}\right]$$

- Expansion:

$$a = \langle l^* \rangle / \lambda_0 \qquad b = 2R/\langle l^* \rangle$$

$$1 - R_M = \frac{1}{10} a b^2 - \frac{1}{48}(1+a) a b^3 + \frac{1}{280}(1+a+a^2) a b^4 + \mathcal{O}[b^5]$$

$$\int_0^r du e^{-u} \left[1 - e^{a(u-r)}\right] = \frac{1 - e^{-ar} - a\left(1 - e^{-r}\right)}{1-a} \approx 1 - e^{-war^2}$$

$w = 0.19$

$$\begin{aligned} 1 - R_M &= \tfrac{1}{5} w a b^2 - \tfrac{3}{70}(wab^2)^2 + \mathcal{O}[(wab^2)^3] \\ &= c_1 A^{2/3} - c_2 A^{4/3} + \mathcal{O}[A^2] \ . \end{aligned}$$

z	c_1	c_2
.25	0.0095	0.000096
.45	0.0103	0.000114
.65	0.0142	0.000217
.85	0.0314	0.001059

A-dependence of absorption

1-R_M=c_1 A^(2/3)+c_2 A^(4/3)

z	c_1	c_2	$\langle l_h(z) \rangle [fm]$
.25	0.0095	0.000096	10.15
.45	0.0103	0.000114	11.72
.65	0.0142	0.000217	12.34
.85	0.0314	0.001059	11.98

Table 1
Computed values of the c_1 and c_2 coefficients in Eq. (26), and average hadron formation time for π^+ production at different z values. For each value of z, we have taken the appropriate average value of ν provided by the HERMES experimental data for a Krypton target [14]. The large value of $\langle l_h \rangle > 10$ fm justifies neglecting hadron interactions with the nucleus for our purposes.

A dependence of attenuation
Energy loss model

$$z = E_h / \nu \rightarrow z^* = E_h / (\nu - \varepsilon) = z / (1 - \Delta z)$$

Quark energy loss ε produces a shift of the hadron momentum fraction z $\rightarrow z^*$

$$\tilde{D}_q^h(z, Q^2; L) = \int_0^{1-z} d\Delta z \, \mathcal{P}(\Delta z) \frac{1}{1-\Delta z} D_q^h(\frac{z}{1-\Delta z}, Q^2)$$

A dependence of attenuation Energy loss model

(hard sphere nucleus $R = r_0 A^{1/3}$)

- Hadron momentum fraction shift:

$$z = \frac{E_h}{\nu} \longrightarrow z^* = \frac{E_h}{\nu - \epsilon} = \frac{z}{1-\epsilon/\nu}$$

- Modified FF:

$$\tilde{D}_q^h(z,Q^2;L) = \int_0^{1-z} d\Delta z\, \mathcal{P}(\Delta z) \frac{1}{1-\Delta z} D_q^h(\frac{z}{1-\Delta z}, Q^2) + p_0 D_q^h(z,Q^2) \qquad \Delta z = \epsilon/\nu$$

$$\tilde{D}(z) = D(z) - \langle \Delta z \rangle h_1(z) - \langle \Delta z^2 \rangle h_2(z) + O(\Delta z^3)$$

$$\langle \Delta z \rangle = \frac{\alpha_s C_F}{2} \frac{\omega_c}{\nu} = \frac{\alpha_s C_F}{4} \frac{\hat{q}}{\nu} L^2$$

$$\langle \Delta z^2 \rangle = (1+\gamma) \langle \Delta z \rangle^2$$

$$h_1(z) = -D(z) - z\partial_z D(z)$$

$$h_2(z) = -D(z) - 2z\partial_z D(z) - \frac{z^2}{2}\partial_z^2 D(z)$$

- Neglect Deuterium:

$$1 - R_M \approx 1 - \frac{\int d^2b \int dy \rho_A(b,y) \tilde{D}(z;L)}{\int d^2b \int dy \rho_A(b,y) D(z)} = \int d^2b \int dy \rho_A(b,y) \left[1 - \frac{\tilde{D}(z;L)}{D(z)}\right]$$

$$1 - R_M = \frac{4}{5} \frac{h_1(z)}{D(z)} \frac{R^2}{\lambda^2} \left[1 + \frac{12}{7}(1+\gamma)\frac{h_2(z)}{h_1(z)} \left(\frac{R^2}{\lambda^2}\right)^2 + O\left(\frac{R^2}{\lambda^2}\right)^4\right]$$

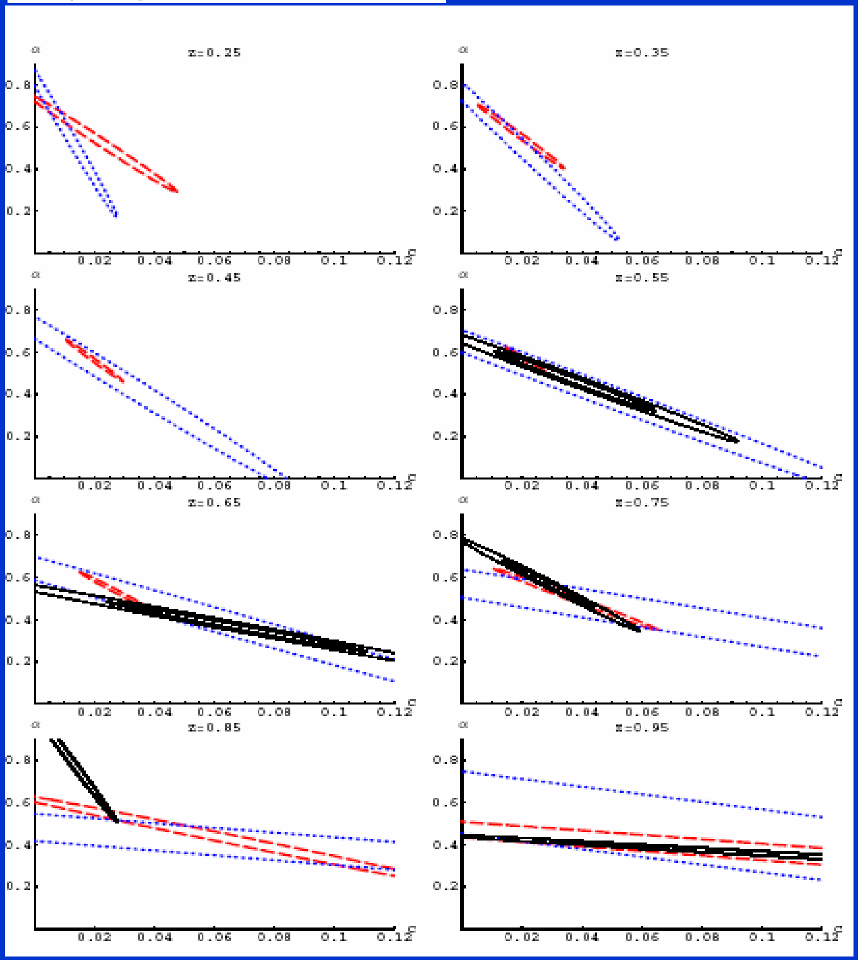
$$\lambda^2 = \frac{4}{\alpha_s C_F} \frac{\nu}{\hat{q}}$$

Results correlate coefficient of A dependence with power of A:

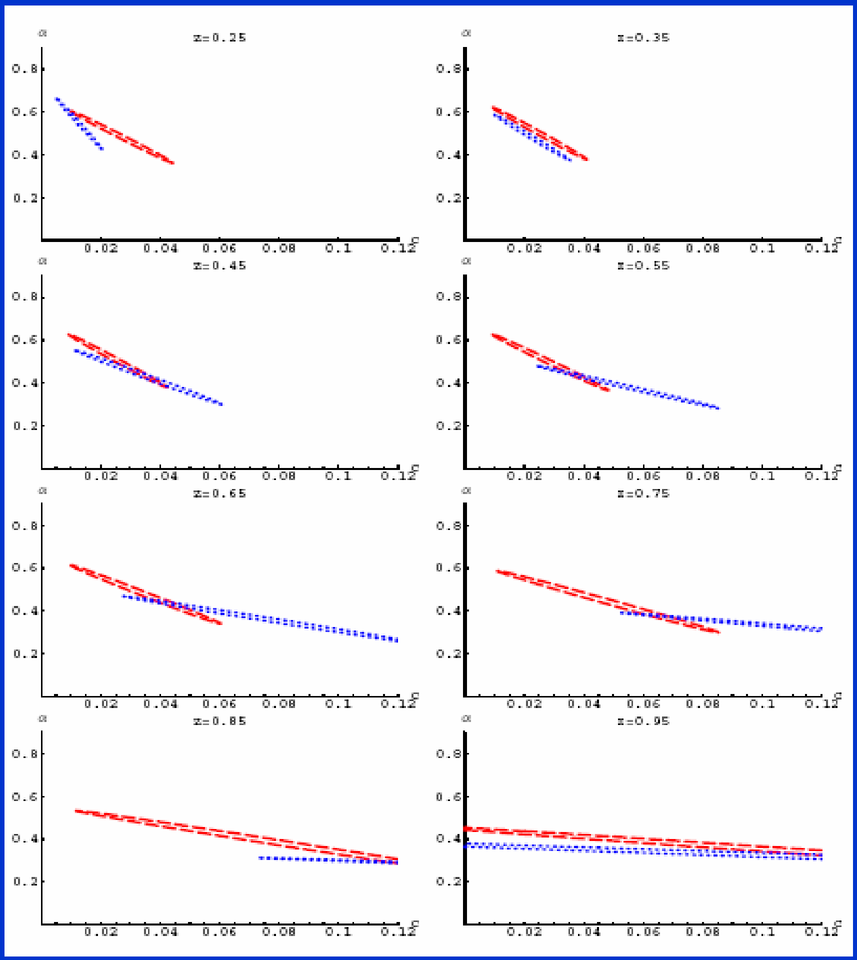
- Correlations can be represented by confidence ellipses
- y-axis=power versus x-axis=coefficient
- Blue=energy loss
- Red=absorption
- Black =data

Mass number A dependence of attenuation

^{4}He, ^{14}N, ^{20}Ne and ^{84}Kr nuclei



^{14}N, ^{20}Ne, ^{40}Ar, ^{84}Kr and ^{131}Xe nuclei



Conclusions:

- Revised absorption model describes data, besides p-production
- Reasonable prehadron (=2/3 of hadron) cross section
- Confidence ellipses at larger z values favour absorption
- Intermediate z values may show energy loss, when more complete data set is available